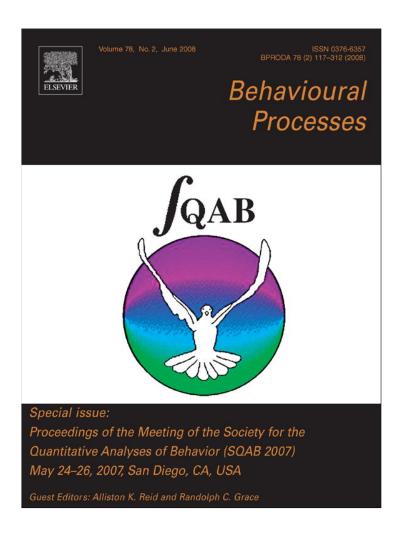
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Behavioral contingency analysis

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Abstract

This paper presents a formal symbolic language, with its own specialized vocabulary and grammar, for codifying any behavioral contingency, including the complex multiparty contingencies encountered in law, economics, business, public affairs, sociology, education, and psychotherapy. This language specifies the "if, then" and temporal relationships between acts and their consequences for the parties involved. It provides for the notation of the probabilities, magnitudes, positive or negative valences, or time delays of the consequences for the parties, and for the parties that would perceive, misperceive, not perceive, predict, mispredict, or not predict events. The language's fractal-like hierarchical and recursive grammar provides for the flexible combination and permutation of the modifiers of the language's four nouns: acts, consequences, time intervals, and agents of acts; and its four verbs: consequate, prevent, perceive, and predict—thereby giving the language the ability to describe and codify various nuances of such complex contingencies as fraud, betting, blackmail, various types of games, theft, crime and punishment, contracts, family dynamics, racing, competition, mutual deterrence, feuding, bargaining, deception, borrowing, insurance, elections, global warming, tipping for service, vigilance, sexual overtures, decision making, and mistaken identity. Applications to the management of practical situations and techniques for doing so, as well as applications in current behavior analysis research and neuroscience, are discussed.

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1. Behavioral contingencies

1.1. What are they?

Behavioral contingencies¹ are the ubiquitous "if, then" situations that influence what people do and do not do. The "if" part specifies some behavior; the "then" part some consequence for the involved party(ies): if you drop a glass (the behavior), it may break (the consequence), and a price label on a product in a store states that if you pay that price, you get the product. The price label contingency might have effects on customer behavior, sales of that product, and competitive pricing by other stores. Further contingencies may be implicit, e.g., if you wait too long, the product may no longer be available.

ever dropped it or picked it up, and the price label contingency is in effect even if no one ever sees the price label or buys anything. In general, however, a behavioral contingency is of practical interest because of its possible behavioral effects.

Behavioral contingencies exist independently of any behavior or dynamics that may occur as a result of their existence.

The dropped glass would break if dropped even if no one

1.2. Behavioral contingencies in human affairs

Behavioral contingencies, often rather complex ones, are a common element of all the behavioral sciences and are at the root of the behavioral phenomena that are of concern in such diverse realms as law, business, education, economics, management, public affairs, therapy, the social sciences, child management and everyday interactions between people.

1.2.1. Prominent examples

• Laws consist, in general, of "if, then" statements of the form, "If a person does or doesn't perform certain

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¹ B.F. Skinner (1969) applied the term behavioral contingency to the "if, then" conditional relationship, "If a response, then a consequence" originally to describe the basic paradigm of operant conditioning, "If a response, then a consequence." The more traditional usage – temporal contiguity regardless of conditionality – is not the sense in which the term is used here.

acts, certain consequences for that person shall follow". Laws are, in essence, behavioral contingency statements intended to regulate, modify, or influence behavior in a society.

- Education systems involve the behavioral contingencies that govern the interactions of students, teachers, parents, administrators, unions, textbook publishers, and members of the community.
- When business managers seek to improve operations by means of incentive compensation systems, work flow systems, and safety practices, they operate on the behavioral contingencies involved.
- The rules of games, ranging from tic-tac-toe to baseball, bridge, poker, or chess, are behavioral contingencies that determine how the games are played.
- Many everyday interactions between people involve behavioral contingency statements of the general type "If you do A, I will do B," including promises, requests, enticements, and threats, sometimes with reference to other parties, time periods, probabilities, or other qualifications.

Unlike the other major determiners of behavior – the personal histories of the involved parties and the immutable realities of physics and biology – behavioral contingencies can be modified and designed.

1.3. The roles of signals and experience

The effects of a prevailing behavioral contingency depend on the individual's history or previous experience with that contingency or with related ones.

Example. The effect of a price tag depends on ability to read and on prior exposure to the product and to price tags. For verbal individuals, a contingency can be communicated by verbal descriptions, including "rules". For non-verbal individuals, a contingency can be communicated by other kinds of signals² and the information conveyed about a contingency can be accurate or inaccurate, complete or incomplete, well understood or only partially understood.

1.4. The value of a formal behavioral contingency language

The present paper presents a formal language, with its own specialized vocabulary, grammar, and syntax, for the systematic codification and analysis of any behavioral contingency. The practical application of this language falls into the province of behavioral technology,³ but given the central place of behavioral contingencies in behavioral science and its applications,

behavioral contingencies are also worthy of study in their own right.⁴

It is true that formal codifications of behavioral contingencies can often also be expressed, sometimes with less effort, by means of natural language, as can the codifications of other formal symbolic languages like those of mathematics or symbolic logic. But in the case of complex behavioral contingencies, formal codification, with the attendant identification of key variables and features, is often a necessary precondition for their systematic study and analysis, whether by means of experiments with living subjects, computer simulation, operations research, or other methods.

Formal symbolic languages, including the present one, also provide the advantage of cutting across all natural languages (universality), with codifications that are succinct and avoid ambiguity. They can reveal relationships among elements that would not be revealed as clearly by natural language descriptions, and in science they can aid in the development of classification systems and theories.

While the antecedent of the present language (Mechner, 1959) was developed as a tool for the specification of independent variables and experimental procedures (Mechner and Latranyi, 1963; Mechner et al., 1963), and has found pedagogic uses like those cited in Section 1.5, the present expanded language is designed for the codification of any behavioral contingency, including complex multiparty contingencies. The language is also well suited for the specification of the situations and contingencies that are the subject matter of behavioral economics (Camerer and Lowenstein, 2003; Becker, 1976, 1995, 1997), including some (described in Section 5) that have become the subject of current behavior analysis research.

Clearly, this type of language cannot generate new knowledge about behavior or make empirical assertions of any kind. Behavioral contingencies do not describe or predict what behavior will actually occur—they only specify consequences for the parties involved *if* some specified behavior occurs. From the standpoint of experimental science, behavioral contingencies have the status of independent variables, and if used as such in experiments, their empirically observed behavioral effects would have the status of dependent variables.

1.5. Related work

While this paper may be the first effort to develop a general language for codifying any behavioral contingency, special-purpose notation systems for behavioral contingencies have been proposed in the past.⁵ The original language and notation sys-

 $^{^2}$ The technical term for "signal" is "discriminative stimulus." The term "stimulus" is avoided here because of its connotation of stimulus-response and reflex phenomena.

³ The application of behavioral contingencies is key in such applied areas as educational technology, behavior management, clinical interventions, and business applications. Sidman (2004) pointed out that the application of behavior analysis to human affairs has still fallen far short of the potential.

⁴ The distinction between independent and dependent contingencies proposed in Weingarten and Mechner (1966), and hierarchical contingency structures, may be examples of types of issues that a science of contingencies might address.

Some behavioral notation systems or paradigms (e.g., Chisholm and Cook, 1995; Findley, 1962; Goldwater and Acker, 1995; Keller and Schoenfeld, 1950; Malott et al., 1993; Mattaini, 1995) have been used to codify behavior analysis principles for instructional purposes and for describing behavioral phenomena that actually occur, as opposed to behavioral contingencies as defined here. These behavioral notation systems have no direct relevance to the theme of the present paper.

tem for codifying behavioral contingencies (Mechner, 1959) dealt with examples drawn mainly from the experimental psychology literature on schedules of reinforcement, escape and avoidance behavior, discrimination learning, and maze learning. Weingarten and Mechner (1966) took an initial step in extending the language to contingencies involving multiple parties.

Skinner (1958) proposed a notation for describing certain specific schedules of reinforcement graphically. Snapper adapted "state notation" (Snapper et al., 1982; Stephens and van Haaren, 1977; Tanaka and Nomura, 1993) for the computer control of certain types of behavioral experiments. In a detailed comparison of the Mechner (1959) notation system and state notation, Michael reported his use of the Mechner language in the undergraduate and graduate courses he taught at Arizona State University (Michael and Shafer, 1995). Michael also used the Mechner (1959) language for his laboratory manual (Michael, 1963), and it was used in the textbook *Behavior Analysis and Learning* (Pierce and Epling, 1995). A strippeddown version of it was used in a laboratory instruction manual (Millenson and Leslie, 1979).

2. The language's vocabulary, grammar, and syntax

2.1. The elements of the language

This paper attempts to show how the proposed language's sparse vocabulary and simple grammar can describe a wide range of complex behavioral contingencies in diverse domains.

The units of any formal symbolic language reflect the conceptualizations of its field: the basic units of chemical notation are atoms; those of musical notation are notes; and those of the present language are acts A, time intervals T, consequences C, and the involved parties (designated here by arbitrarily chosen lower case letters).

These are the language's six basic elements:

- 1. $A \rightarrow$, read as, "If act A occurs then ...". Thus, every A is understood to be preceded by an implied "If".
- 2. Agent of the A: The agent(s) of an act A are designated by arbitrarily chosen letters placed in front of the A (e.g., aA, bcA).⁸
- 3. $T \rightarrow$, read as "upon termination of time T...". The time period T is part of the situation, as opposed to a property of the behavior.
- 4. A consequence C is any situation or event resulting from an A→ or a T→. The term refers broadly to all relevant aspects of the prevailing environment.
- ⁶ The term "act" is used here, rather than "response," which has the undesired connotation of a stimulus-response reflex, and usually of a brief response performed by a single individual.
- ⁷ The term "consequence" is used in favor of the traditional term "stimulus," again to avoid the "stimulus-response" connotation. Consideration was given to using *S* for "situation," but the term consequence conforms better to behavior-analytic thinking.
- ⁸ The term "agent" or "party" refers to an individual, group, team, company, populace, or country that would perform the designated act.

- 5. *Prevention of a consequence*: A consequence is prevented when a horizontal arrow leading to it is cut by a vertical arrow originating from another *A* or *T*, like this: $\overrightarrow{\bot}$.
- 6. Simultaneity of onset: A bracket enclosing vertically listed As, Ts, or Cs indicates simultaneity of onset of the listed conditions. For instance, C would be read as "When C is present then if A, then . . .". The order in which the conditions are listed has no significance.

Only As or Ts can consequate 9 Cs or further $A \rightarrow$ or $T \rightarrow$ contingencies. Thus, arrows can never emanate from a C, only from an A or a T.

2.2. Architecture of the grammar: the four quadrants

Every element of the language can have up to four modifier categories, as shown in the four quadrants of each symbol in Fig. 1. Each quadrant is allocated to one of the four modifier categories in a consistent way. The modifiers can, in turn, have the same modifiers in the corresponding quadrants, in a theoretically infinite hierarchical and recursive regress, fractal mode. 10

The subscript (lower right quadrant, e.g., the numeral 5 in aA_5) is an arbitrary numeral indexed to a legend where the referenced entity is described (see Section 2.3).

The superscript (upper right quadrant) is allocated to the entity's attributes, which can be viewed as the language's adjectives and adverbs—probability, magnitude, valence, ¹¹ duration, or variability, as explained in more detail in Section 2.4.

The pre-subscript (lower left quadrant) of an entity indicates the party(ies) that would *perceive*¹² the entity (see Sections 2.5–2.8), and the pre-superscript (upper left quadrant) indicates the party(ies) that would *predict*¹³ the entity (see Section 2.9). One of the theses of this paper is that the verbs *perceive* and *predict* are a unique (necessary and sufficient) and parsimonious key to the generality of the language's applicability.

2.3. Subscript: indexing to a legend

Arbitrary subscripts (usually numerals) can index the diagram's symbols to descriptions in a legend. They can also provide differentiation between different As, Cs, or other enti-

⁹ The verb *consequate* has been used in behavior analysis and is useful in the present context. The alternative verbs "produce," "generate," "result in," or "cause" tend to carry undesired epistemological baggage.

¹⁰ These are also characteristics of such formal languages as symbolic logic and mathematics, as well as of most natural languages.

¹¹ A *C*'s valence indicates whether the *C* is rewarding or punishing, desirable or undesirable, beneficial or harmful, positively or negatively reinforcing.

¹² The verb "perceive" is understood to encompass all of the learned or genetically encoded discriminative responses to the stimuli or situation comprised by the perceived entity.

¹³ The verb "predict," as used here, refers only to the behavioral effects of the party's prior contact and experience with the contingency involved, or with the signals (verbal or non-verbal discriminative stimuli) that were previously associated with the contingency. No other connotations are implied.

F. Mechner / Behavioural Processes 78 (2008) 124-144

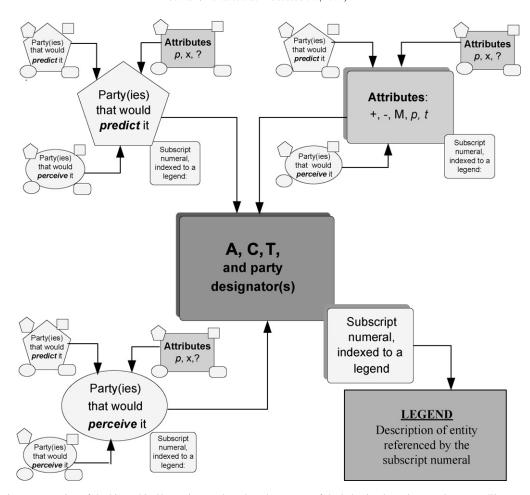


Fig. 1. Diagrammatic representation of the hierarchical/recursive quadrant-based grammar of the behavioral contingency language. The center box containing the language's four nouns has four quadrants, shown as shapes that represent modifier entities (e.g., perceive, predict, attributes, index numbers to the legend), each of which in turn has its own analogous quadrants.

ties. For example, to indicate that the duration of C_1 is T, a new consequence C_2 is shown replacing C_1 : $\begin{bmatrix} \mathbf{T} \rightarrow \mathbf{C_2} \\ \mathbf{C_1} \end{bmatrix}$.

When descriptions consist of one or two words, it may be convenient to show those words in the subscript positions within the diagram, while lengthier descriptions can be provided in the legend. In a_7A_5 , the legend might describe the nature of party a as a particular individual, team, population, or government under the arbitrary numeral 7, and the nature of act A under the numeral 5. Without subscripts and legends, every diagram would represent the infinite number of contingencies that have the same structure. Specificity is achieved only by the use of descriptive subscripts.

2.4. Attributes of entities

2.4.1. Valence

The notation C^{a+} or C^{a-} means that C has a positive or negative valence for party a. $C^{a+,b-}$ means that the valence is positive for a and negative for b, and C^{ab-} means that the negative valence applies to both. $C^{a(\pm)}$ means that the valence for a can be either positive or negative.

2.4.2. Magnitude M

The magnitude attribute for act A, as in A^M , can refer to such intensive dimensions as effort, force, loudness, duration, or an amount of money. When M refers to the act's duration, a lower case t replaces the M. The duration t of an act is measured from its beginning to its termination. In $A_{\text{start}} \rightarrow A_{\text{end}}^t, A^t$ is read as "end of act A's duration." The term A_{start} can be omitted, since every act that ends must also have started.

The duration attribute is particularly significant when an act is repetitive and can continue or end at any time. Examples of such acts, often called activities, are walking, running, waiting, foraging, working, or practicing a skill. In behavior research, a latency or inter-response time would be codified as an act whose terminating event is A^t .

The magnitude of a *C* would usually refer to the magnitude of its positive or negative valence, but can also refer to any scalable dimension like value or amount of money, or to some qualitative dimension.

For time periods, T^M refers to the duration of T.

A magnitude of a consequence or of a valence that changes as a function of some variable x can be represented

by $M = f(x_i)$ shown in the attribute position of the consequence.

2.4.3. Probability

A lower case p, or an actual value of a probability shown in an entity's upper right quadrant, indicates the probability, in the analyst's view, that the entity would occur.

2.4.4. Variability

 T^{V} or M^{V} indicate that the modified entity is variable.

2.4.5. Negation

The tilde sign \sim , representing the negation attribute "not", is placed above the entity's symbol. ¹⁴

Although these attributes are fully sufficient for codifying the wide and diverse range of situations analyzed in this paper, additional attributes may, in the future, be required to meet challenges presented by specialized applications in other fields like law, business, economics, sociology, public affairs, or psychotherapy.¹⁵

2.5. The notation of perception

In $A \to {}_{ab}C$, the lower left quadrant of C indicates that both parties a and b would 16 perceive the consequence C of act A. To show that party a would not perceive C but that b would, the notation is $A \to {}_{\tilde{a}b}C$, with the a having a tilde sign above it meaning "not a".

 $aA \rightarrow_{\tilde{a}b} T \rightarrow_{ab} C$ is read as, "If party a does A, then C occurs after time lag T. The C would be perceived by both a and b, and the time lag would be perceived by b but not by a." The T would be shown without any pre-subscripts if the analyst considers perception of the time lag not relevant to his focus.

The contingency $aA_1 \rightarrow bA_2 \rightarrow$ is read as "If $aA_1 \rightarrow$ then if $bA_2 \rightarrow$ then ...". In the case where the desired meaning is "If $aA_1 \rightarrow$ then bA_2 would occur," the bA_2 term would have the status of a consequence C.

2.6. The notation of misperception

 ${}_aC$ implies that party a's perception of C would be "correct" in the sense that it corresponds to the analyst's belief, that being the sense in which the terms "correct" or "actual" will be used throughout. If the analyst wishes to show that the perception would be a "misperception" or idiosyncratic, 17 the misperceiving party's designator, still shown in the lower left quadrant, would have the letter x in its own upper right (attribute) quadrant. For example, a^xb^C means that a would misperceive C and that b would perceive it correctly. To elaborate on the misperception, the analyst could explain under, say, the

legend's numeral 5, $a_5^x b^c$ that a would, for example, misperceive a neutral comment as hostile. The term "misperceives" is used broadly here for any perception that is idiosyncratic or "subjective," in the sense that it deviates from the analyst's belief.

When time is an attribute of an act (i.e., its duration) rather than a feature of the situation, the party's perception, misperception, or non-perception of the act's duration would be shown as the pre-subscript of the t, like this: $\mathbf{a}\mathbf{A}^{ax}^{t}$ or $\mathbf{a}\mathbf{A}^{\tilde{a}t}$. The latter diagram could mean "a not keeping track of how long its act A was taking."

2.7. Perception, misperception, and non-perception of valences

If a party would perceive (or misperceive or not perceive) a consequence C, would that party then also perceive (or misperceive or not perceive) that same C's valence? The answer is that it would, based on the distributivity rule which states that any perception, non-perception, or misperception of the C applies also to that C's valence and any of its other attributes.

Example. $aA \rightarrow {}_{ab}C^{b-,\,a+}$ could mean that a would harm b and benefit himself, with both a and b perceiving this consequence and its valences. If either a or b would not perceive either or both of these valences, that party's designator would be shown with a tilde over it, in the lower left quadrant of the non-perceived valence term, to override the distributivity rule. For example, to show that a would not perceive the C's valence b- in spite of perceiving the C itself, the b- valence symbol would need the

pre-subscript \tilde{a} . ab $\tilde{a}b^{-}$, a^{+} would mean that a would perceive *neither* C nor its valence.

To show that a party would misperceive a C's valence but not the C itself, that party's designator, with the letter x in its upper right quadrant, is shown in the valence's lower left quadrant. Familiar examples are Adam and Eve, or Snow White, who might have perceived the apple correctly but misperceived its valence.

¹⁴ It is more convenient to place it above the entity than in its upper right quadrant, which is where it would be placed based on the placement rule for attributes. In symbolic logic it is usually placed in front of the character, but here this placement would have the drawback of the multiplicity of meanings (including "approximately") that such a placement has in various other contexts.
¹⁵ It may be possible, for example, to extend the language to the description of the environmental contingencies that result in speciation and evolution—the adaptation of species to contingencies imposed by the environment. The consequences would then be characteristics like camouflage or immunity, and the valences might pertain to competitive advantages or disadvantages related to reproduction and survival. The language may also be applicable to the contingencies involved in the evolution of cultural practices (Glenn and Madden, 1995; Glenn, 2003, 2004), and perhaps to any system involving consequation with feedback.

 $^{^{16}}$ The term "would" or "will" is used throughout, to maintain consistency with the conditional "if, then" logic of contingency statements, e.g., "If act A, then party(ies) a, b, c would or will perceive or predict a designated entity." A statement of the form that a, b, or c actually do perceive or predict an entity, without the qualification of conditionality, would be inconsistent with the definition of a behavioral contingency statement.

¹⁷ In behavior analysis terminology, misperception or idiosyncratic perception means responding to a stimulus (here termed consequence) as if it were a different stimulus.

The valence shown is the one the analyst considers the real one. The precise nature of the misperception he has in mind can be described in a legend. Here are some combinations (remember that the *C*'s pre-subscript applies not only to the *C* but also to its attributes):

 ${}_{a}\mathbf{C}^{a}$ a perceives both the C and its negative valence for itself.

 $\mathbf{a}^{\mathbf{C}^{\mathbf{a} \times \mathbf{d}^{-}}}$ a perceives the C and misperceives its negative valence.

 $a^{-1}a^{-1}$ a misperceives both the C and its negative valence.

 ${\bf a}^{\tilde{\bf a}^{\bf a}}$ a perceives the C but not its negative valence.

 $\tilde{a}^{C^{a-}}a$ does not perceive either the C or its negative valence.

2.8. Misperception of a party: who done it?

baA means that b perceives that a is the agent. $b^x aA$ means that b misperceives the fact that a is the agent. This distinction could come up in false accusations or in criminal law where there can be divergent assertions regarding the agent of an act. Again, the nature of the misperception could be described in a legend. Analogously, the notation $a^x T$ means that a would misperceive T, and the notation $\bar{a}T$ means that a would not perceive T at all.

2.9. Prediction

The term "predict," as used here, encompasses all of the behavioral effects of the party's prior experience with similar contingencies, or with signals (called verbal or non-verbal discriminative stimuli by behavior analysts) that were previously associated with similar contingencies. Signals consisting of verbal statements of contingencies are sometimes called rules. When the analyst indicates that a party would predict a consequence, he therefore reflects his beliefs regarding the party's relevant history.

A signal that might occasion a party's prediction of a consequence has the status of a *C*.

2.10. The notation of "predict"

The party(ies) that would predict a C are designated in the C's upper left quadrant, like this: ${}^{a}C$. $aA \rightarrow {}^{a}_{a}C$ means that a would perceive C when it occurs and would also predict it. $aA \rightarrow {}^{a}_{\tilde{a}}C$ means that a would predict C but would not perceive it (note the \tilde{a} in C's lower left quadrant).

Examples. Committing suicide, *predicting* the consequence of sending a letter without *perceiving* that consequence when the letter reaches its destination, predicting the consequences of issuing an order without perceiving the order being carried out, dropping a bomb on a target. The reverse, $\partial A \rightarrow \tilde{\partial} C$, means

that a would perceive C when it occurs but would not have predicted it. Here one might say that C would surprise a.

Depending on the desired emphasis and focus, the analyst may or may not indicate the party(ies) that do or do not perceive or predict a particular consequence.

2.11. Perceiving or predicting another party's perception or prediction

 $aA \rightarrow {}^bC^{a-}$ means that party b would predict that a would hurt himself. The notation ${}^{ax}\mathbf{C}^{a-}$ means that a would mispredict it. To show that b would also *perceive* that a was mispredicting C^{a-} , the b would be shown in the lower left quadrant of the a^x , like this ${}^{bax}\mathbf{C}^{a-}$. The diagram ${}^{bax},{}^{b}\mathbf{C}^{a-}$ would show that b would, in addition, predict correctly that a's act would be hurtful to a.

When the verb predict is applied to a probability, the meaning is similar to that of the verb "estimate." The expression ${}^{a}p_{1}$, for instance, would be read as "party a's prediction/estimation of p_{1} ."

2.12. Nuances of meaning involving probabilities and predictions

This diagram could mean that if a asks (aA_1) b for a favor, then if b complies (bA_2) with the request ${}_bC_4$, the consequence $aA_1 \longrightarrow bA_2 \longrightarrow C_3$ would be C_3 . The analyst can express the following nuances regarding bA_2 :

 $(bA_2)^p$: The probability of bA_2 occurring is less than one. b^pA_2 : p is the probability that b (rather than another party) would do A_2 .

 $^{a}(bA_{2})$: a would predict bA_{2} . Replacing the a with a^{p} would mean that the analyst considers the probability to be p that a would predict bA_{2} .

2.13. Why the terms "aware," "intend," and "expect" are not used

The term "aware," when applied to a contingency, is imprecise in that it could apply equally to the act only, to the consequence only, or to the party's prediction of the consequence only, and when applied to the act or to the consequence it does not distinguish between its perception and prediction. The terms *perceive* and *predict*, on the other hand, are precise in that they can be applied with pinpoint precision to acts, consequences, and attributes, and unlike the term "aware," they can be modified by a probability or as a *misperception* or *misprediction*.

The terms "intend" or "expect" are also imprecise in that they do not tell us if an intent or expectation is still present if the consequence is modified by a probability or delayed by a time interval. The contingency language avoids such ambigu-

¹⁸ For other discussions of related terminological issues see Chisholm (1957), Baum and Heath (1992), Foxall (2007) and Hineline (2003).

ity. It is able to express these concepts fully, along with their many possible nuances of meaning, by specifying that the act's agent *predicts*, *mispredicts*, or *does not predict* the act's consequence, with further nuances provided by recursively assigned modifiers.

2.14. Vertical arrows that terminate and change contingencies

A vertical arrow (initiated by an A or a T) cutting a horizontal arrow, like this, T terminates the contingency represented by that horizontal arrow. This diagram could describe the contingency involved in catching a plane or train or in meeting a deadline.

To show that a vertical arrow not only prevents a consequence but also creates a new one, the vertical arrow is shown pointing to the new consequence C_2 , as in the diagram on the left.

$$\begin{bmatrix} \mathbf{C_1} & \tilde{a}^b \mathbf{C_2}^{b+, a-} \\ a\mathbf{A_3} & \xrightarrow{a_b} \mathbf{C_5}^{a+} \\ b\mathbf{A_4} & \end{bmatrix}$$

Note that at the intersection of the arrows, only the contingency represented by the horizontal arrow is terminated or changed. The horizontal arrow does not cut or affect the contingency represented by the vertical arrow.

2.15. Recycling contingencies

To show that a contingency remains in effect and can repeat, a recycling arrow is used. For example, when a CD has been played, it can be played again. To indicate that an act can recycle n times, the n is written above the recycling arrow: $A \to C$.

3. Other features of the contingency language

3.1. Distributivity relations among the quadrants

The default rule is that an entity's subscript (the numeral that indexes a legend item) is distributive across all of the entity's modifiers, but the analyst may sometimes want to use separate subscripts for individual modifiers.

The verbs *perceive* and *predict* also extend to the modified entity's attributes (shown in the upper right quadrant)—i.e., any entity that is perceived or predicted includes all of that entity's attributes, these being similarly perceived or predicted.

For example, $b^{ap}C^{a+}$ means that b would perceive that p is the probability that a would perceive C and C's valence.

All other relations are non-distributive. For example, in b **C** a would not also perceive the fact that b would predict C, that is, the perception by a would not be distributive to the upper left quadrant, and vice versa. To show that a would also perceive that b would predict C, the a would have to be shown again in the b's lower left quadrant.

3.2. The grammar of consequences

A general default feature is that only one consequence C is present at one time, because every C is presumed to include all of the relevant features of the prevailing situation. Thus, any change of C is a new, again all-inclusive, C produced by a further A or T.

Suppose, for instance, that A_1 consequates C_{tone} , and C_2 $A_1 \rightarrow C_2$ C_{tone} is a light coming on subsequently. C_2 then replaces C_{tone} (tone alone), and may then consist either of both the light and the tone together or of the light alone. The legend would state which, if the analyst considers the distinction relevant. In either case, C_{tone} (tone alone), having been replaced by C_2 , is no longer present.

3.3. When all consequences are represented by one C

The analyst would always narrow his analysis to those consequences that are relevant to his focus and would represent these with a single *C*, notwithstanding the fact that all real-life acts have multiple and innumerable consequences, in the sense of the proverbial wing flap of the butterfly in China ultimately affecting the weather on the other side of the earth. A party performing an act would never perceive or predict all of the act's consequences.

A mundane example. If I open the refrigerator and pour myself some juice, I would probably predict that I would be drinking juice in a few seconds and then perceive it. I would probably not perceive or predict all of the physical, chemical, and thermal consequences of opening the refrigerator or of the juice on my stomach chemistry.

A weightier example. If a company's directors close down a factory, they would predict and/or perceive certain of the consequences and their time lags, but not numerous others.

3.4. Diverse consequences: when more than one consequence is shown

On the other hand, when the modifiers of the consequences are heterogeneous and yet relevant to the analyst's focus, a single *C* cannot always represent all of them. The analyst would then

represent them with more than one
$$C$$
:
$$aA_1 \rightarrow \begin{bmatrix} bC_2 \\ cC_3 \end{bmatrix}$$

Examples. (1) Party a introduces parties b and c to each other. The result would be two consequences— ${}_bC_2$ (b's perception of the situation that includes party c), and ${}_cC_3$ (c's perception of the situation that includes party b). Also, C_2 and C_3 may have different valences for the three parties a, b, and c, which may also have different predictions or perceptions of those valences; (2) a business executive a assigns a task to b and c. When b and c divide the work and each takes a different part of the job, the consequence for each of them would necessarily be different.

In some cases, the analyst may want to distinguish between diverse consequences of a single act.

Example. Shooting at a target and hitting it. In this case, A_1 is pulling the trigger, C_1 is the gun firing, and C_2 , is the bullet's point of impact. A party could perceive and/or predict either of these consequences and not the other.

There are also cases where the analyst may want to $aA_1^{p_3}$ $bC_2^{p_4}$ bA_3 C_4 assign different *probabilities* to the performance of the act and to its consequence occurring (e.g., to the act of shooting at a target and to the *probability* of hitting it).

In the real world, the flow of acts and their consequences is continuous and seamless. In codifying a situation, exactly where in that flow the analyst chooses to draw the demarcation line between an act and its consequence depends on the desired focus. When the consequence of aA_1 serves as the cue for party b's act, this cue can be just the sight or some other consequence of a performing the act.

3.5. Consequation by external agencies

A situation C may be consequated by an external agency e as well as by one of the parties involved in the contingency proper.

Examples of externally consequated *Cs.* The hand that a bridge player is dealt, a test item presented to a test taker, or any situation presented by the physical environment. In these cases, the external agency would be the card dealer, the presenter of the test item, or the physical environment. Acts by such external agencies *eA* would be shown only if the analyst considers this aspect of the contingency relevant to his intended focus.

The diagram at the left shows that a would perceive that he has an e-mail, ${}_{a}C_{3}$, sent by an unknown external agency eA_{1} .

 $\mathbf{eA_1}$ $\mathbf{A_2}$ $\mathbf{A_5}$ $\mathbf{A_5}$ $\mathbf{A_6}$ $\mathbf{A_6}$ $\mathbf{A_6}$ $\mathbf{A_6}$ $\mathbf{A_6}$ $\mathbf{A_6}$ $\mathbf{A_6}$ $\mathbf{A_6}$ $\mathbf{A_6}$ If a opens the e-mail (aA_2) , a will predict and perceive its display C_4 on the screen, including an attachment. The a shown in C_4 's upper left quadrant means that a would predict that opening the e-mail would consequate the screen image ${}_aC_4$. This a pertains only to C_4 and has no bearing on the $aA_5 \rightarrow C_6^{a-}$ contingency, namely on whether a would also perceive or predict that if aA_5 (clicking on the attachment), the computer would be infected with a virus C_6^{a-} . The \tilde{a} designator in the upper left quadrant of \tilde{a} C_6^{a-} indicates that a would not predict that opening the attachment would incur a virus. To show that a would predict it, the a would need to be shown in the upper left quadrant, like this, a C_6^{a-} , without the tilde.

3.6. The notation of uncertainty

Uncertainty is indicated by a question mark after the entity to which the uncertainty applies. The specific nature of the uncertainty, or the reason for it, can be elaborated in the legend. The numeral indexing the legend can be shown in the question mark's lower right quadrant: b?₅.

a? \mathbf{C} means that the analyst is uncertain that a would predict C. $(a?)b^{\mathbf{C}}$ means that the analyst is uncertain as to whether a would perceive the C. When a party, rather than the analyst, is uncertain, the party's designator is shown in the question mark's attribute quadrant: $(a?^a)b^{\mathbf{C}}$. (The logic underlying this rule is that the uncertain party is like an attribute of the uncertainty.) To show that the analyst is uncertain regarding the *correctness* of a perception or prediction, the question mark is shown in the party's attribute quadrant: a? a own prediction, an additional a is shown in the question mark's attribute quadrant: a? a own prediction, an additional a is shown in the question mark's attribute quadrant: a? a or a of a is uncertain as to the valence for a but not for a, the notation is a0.

3.7. Uncertainty and probability

When to use the question mark and when to use the probability notation: a question mark indicates the lack of some existing information where the uncertainty would be eliminated if that information were provided. An indication of a probability, on the other hand, suggests that the event depends on an external process like the tumbling of dice, the order of the cards in the deck, or the results of decision processes of others. In such cases, a second branch of the consequating arrow can show the consequence for the 1-p complement of the probability, if doing so is relevant to the focus. A probability can have as a modifier a party's prediction/estimation (shown as the p's pre-superscript), but an uncertainty cannot.

Notations of uncertainty or probability are used only when they are relevant to the analyst's focus. Although all consequences would necessarily be probabilistic to some degree, relatively small uncertainties are usually unimportant in relation to the focus, and the analyst may therefore not indicate them.

3.8. Simplifying assumptions and abbreviations

Formal symbolic statements are inherently stripped down to presumed essentials through the use of simplifying assumptions intended to eliminate features that might mask the intended focus. With that intent, the analyst simplifies the contingencies being represented by selecting some features and omitting others. The result bears the same type of relationship to real-life contingencies that a drawing of an object bears to the real object.

A common simplifying assumption is the omission of the Ts representing the time lags between all acts and their consequences. When the time lag is relevant, the contingency might be shown as $A \rightarrow T \rightarrow C$.

Abbreviations provide another way to simplify diagrams. For example, the symbol C^{a+} is an abbreviation. If the analyst considers the specific reasons for the valence being positive for a important, he can elaborate them in the diagram proper or in a separate diagram indexed to the point of insertion.

Examples of reasons. Upon the occurrence of C, a might be able to avert an impending negative consequence or procure a positive consequence by certain further acts.

3.9. "And" relationships: cooperation, contracts, agreements

When a consequence depends on the occurrence of all of two or more events, the symbolic logic symbol \cap for the intersection of sets (the "and" relationship) is used.¹⁹

Example. Mother says to child, "I will read you a story (C^+) in 5 min (T_3) if you have brushed your teeth (A_1) and gotten into bed (A_2) in those 5 min": $(A_1 \cap A_2 \cap T_3) \rightarrow C^+$. The \cap s mean that all three conditions must be met.

The \cap symbol is also used in codifying cooperation, as when a and b perform separate acts aA_1 and bA_2 when they cooperate.

Example. If a and b make an agreement $(aA_1 \cap bA_2)$ by exchanging promises, undertakings, signatures, or money, and agree to perform further acts $(aA_3 \cap bA_4)$, namely the undertakings stated in the agreement, presumably for the parties' mutual benefit C^{ab+} , this would be represented as $(aA_1 \cap bA_2) \rightarrow (aA_3 \cap bA_4) \rightarrow C^{ab+}$. The specific terms of the agreement can be elaborated in the legend. If aA_1 and bA_2 are thought of as a single act rather than as separate acts, the notation abA would be used instead of the expression with the \cap symbol.

3.10. Durations of acts

In many contingencies the consequence is a function of the amount of time that has elapsed or that is consumed by an act. A Chinese proverb relates the contingency of being afraid to dismount the tiger that keeps getting hungrier. Here t is the duration of $aA_{\rm dismounts}$ (time since mounting), and M and/or p are direct functions of t, where M is the magnitude of the negative consequence and p is its probability.

$$aA_{\text{mounts}} = \begin{bmatrix} aA_{\text{dismounts}}^t \\ C_{\text{riding}} \end{bmatrix}^{t} C_{\text{pain}}^{t,p}$$

Examples. Procrastinating having a dental or medical problem taken care of, fear of reporting a worsening business situation to one's shareholders, fear of confessing a crime, delaying paying a tax penalty, or not fixing a leaky roof.

In the analogous contingency where the valence is positive, patience is rewarded, or the magnitude or probability of the reward increases the longer one waits.

$$aA_{\text{request}} = \begin{bmatrix} aA_{\text{waiting}}^t & C_{\text{reward}}^{a(+)M, p} \\ C_{\text{promise}} & C_{\text{reward}}^{a(+)M, p} \end{bmatrix}$$

Foraging, too, can be described by this general contingency, where finding food may depend on the time t spent foraging.

3.11. Repetitive recycling of an act: learning and practicing

Familiar examples of possible behavioral effects of such recycling contingencies are the progressive decrease in t (i.e. an

increase in the speed of the act) $\rightarrow A^{t} \rightarrow C$ seen in learning and practicing, or an increase in t seen in fatigue or drug effects. Such observed effects can then be described by the function $t = f(n_i)$.

3.12. Behavioral history factors

Some aspects of a behavioral contingency description are independent of the histories of the involved parties, and some are not. The specification of the acts, the time periods, the consequences, the parties, and certain of the entities' attributes like probabilities and magnitudes, reflect the analyst's beliefs about the situation's objective or physical features, without consideration of the parties' histories. But such modifiers as perceive, predict, and the valences of consequences do depend on the parties' histories and reflect the analyst's beliefs about the effects of those histories. For example, the glass will break if dropped regardless of any party's history, but a party's prediction of that consequence, and its valence for that party, would depend on the party's history. Thus, parties' histories can be part of the contingency. The representation of any given situation would therefore depend not only on the analyst's focus and analytic intent but also on his beliefs regarding the parties' histories and motivations, and would, accordingly, be different for different parties, and for the same party(ies) at different times.

4. Examples of applications

4.1. Wrong predictions: getting swindled, wrong number, "friendly fire"

A party might mispredict the consequence of its act A.

Examples. Dialing an incorrect phone number in error, paying a swindler for nonexistent real estate, and a soldier shooting one of his own men. These situations would all be represented as $aA \rightarrow a^x C^{a-}$. The a^x in the C's upper left quadrant shows that a would mispredict C^{a-} . These examples make the point that only consequences that would actually occur (i.e., in the analyst's view) – calling the wrong party, obtaining an invalid deed, and shooting a comrade – can be codified, in this case represented by the a-. Consequences that would *not* occur (e.g., "expectations"), cannot. Only the legend would state what the parties may have intended or expected—calling the desired party, obtaining a valid deed, and shooting an enemy. The diagram can show only

¹⁹ As explained in Mechner (1959), the "and" symbol is an abbreviation for a diagram that would show the possible occurrence of any of the permutations of the sequence of the events in question, with reciprocal vertical arrows showing that the occurrence of each sequence prevents the subsequent occurrence of the others.

that the party would have predicted a consequence that differs from the actual one.²⁰

To show that a would also perceive the actual consequence if and when it occurred in each of the above examples, the a would also be shown in the lower left quadrant of the C, as in $aA \rightarrow {}^{a^{x}}_{a}C^{a^{-}}$

4.2. Unpredicted effect on another party's behavior

When a party a misuses a word in an unfamiliar language or gives someone faulty driving instructions, the consequence would be an unpredicted effect on the behavior of another party. When a word is misused, the consequence might be puzzlement by the other party. When faulty driving instructions are given, it might be the other party getting lost and complaining. In both cases, the actual consequence is negative for a.

If aA is uttering the wrong word and

$$aA \xrightarrow{\rightarrow} bC_{\text{wrong word}} bC_{\text{wrong word}}$$
 $\underline{b}^{ax}A_{\text{"huh?"}} \xrightarrow{\rightarrow} aC_{\text{sound of "huh"}}^{a-}$

 $C_{\text{wrong word}}$ is the utterance of that word perceived (heard) by b, it is b's puzzled reaction "huh?" to the word that would constitute the negative consequence for a, not $C_{\text{wrong word}}$.

If the listener b nods in spite of puzzlement (see diagram)

$$aA_{\text{utters}}$$
 $bC_{\text{wrong word}}$ $bA_{\text{nod}} \rightarrow ba^{x}C_{\text{sight of nod}}^{a+}$, a might misperceive the nod as indicating that b understood. The consequence of the nod is positive for a because of a 's misperception of it. If b predicted a 's misperception, as indicated by the b in the upper left of the a^{x} , the nod would constitute a form of deception by b .

4.3. When nuances determine the meaning: setting a trap

In this diagram, C_3 's valence is negative for b.

$$aA_1 \rightarrow bA_2 \rightarrow bA_2 \rightarrow bC_3^b$$

The two negation symbols \tilde{b} show that b would not predict C_3 or perceive the warning signal C_4 . This could be described as a setting of a trap for b. The a in the upper left of C_3 shows that a would predict that bA_2 would consequate \mathbf{C}_3^{b-} and would therefore have set the trap "intentionally," as one might say. Additional nuances or emphases could be indicated by showing a's in the upper left quadrants of either or both \tilde{b} s (in C_4 and C_3), to show that a would predict b's non-perception of C_4 and non-prediction of $\mathbf{C_3}^{b^-}$. Nuances like these can assume great significance in legal matters.

4.4. Probability of a consequence: creating a hazard

The assignment of probability p to C_3^{b-} , as an additional attribute, defines the contingency as a hazard. $aA_1 \rightarrow \begin{bmatrix} a b & A_2 \\ aC_4 \end{bmatrix}$ The a in the upper left quadrant of

$$aA_1 \rightarrow \begin{bmatrix} ab^aA_2 \rightarrow baC_3^{b-,p} \\ aC_4 \end{bmatrix}$$

The a in the upper left quadrant of b means that a would predict that b would be the agent. The a in the upper left quadrant of A_2 means that a would predict A_2 regardless of who does it. The b in the upper left quadrant of C_3 , in addition to the a, means that both a and b would predict that A_2 , if it occurs, will consequate C_3 and its attributes, which include p. Would a and b then also be predicting that C_3 's probability is p? The distributivity rule (that the modifiers perceive and predict apply also to the entity's attributes) says that they would. An $\tilde{a}p$ in C_3 's attribute quadrant would override the distributivity rule, meaning that a would then not predict p. $\tilde{a}^b p$ would mean that b still would.

4.5. Theft

If b takes the cookie out of a's lunch box (bA_4) before a does (aA_3) , a would be prevented (vertical cutting arrow) from doing

$$\begin{array}{c}
C_1 & \tilde{a}^b C_2^{b+, a-} \\
aA_3 & \xrightarrow{b} a_b C_{\text{cookie}}^{a+} \\
bA_4 & \xrightarrow{a} \end{array}$$

(The fact that the cookie is in a's lunch box in the first place (C_1) would have been consequated by the act of an external agent, eA, not shown.) Since only b would predict that b will end up with the cookie (C_2) , and since \tilde{a} shows that a would not perceive that the cookie is in b's possession, bA_4 would represent a theft. This example makes the additional point that a rigorous definition of the concept of theft, especially from a legal standpoint, requires specification of the perceptions and predictions of both parties, and the valences of the consequences.

4.6. Applications in therapy

To achieve desired results, therapists (psychotherapists, psychoanalysts, behavior therapists, cognitive-behavior therapists, coaches, or counselors) often seek to understand the contingencies involved in the patient's interactions with others (Erickson and Achilles, 2004; Wachtel, 1997). Such contingencies can be generated in part by the interactions themselves, including their reverberations at higher levels of the recursion hierarchy.

Family therapy in particular is an area where modifiers of modifiers can be important. In dealing with family dynamics, the therapist may analyze the contingencies that parents, siblings, and related parties create for each other. These often involve multiple levels of regress of each other's perceptions, mis-/nonperceptions, predictions and mis-/non-predictions of acts and their consequences, as well as of their valences and magnitudes. Since all of these are the result of the parties' histories, one of the therapist's challenges is to modify the effects of such histories,

 $^{^{20}\,}$ When the act's agent would also predict the act's consequence, one might say that the consequation is intentional. However, as was explained in Section 2.13, terms relating to intentionality, as well as expectations, hopes, visualizations, goals, etc., are not needed.

134

an endeavor in which a detailed behavioral contingency analysis can be useful.

4.7. Consequence of omitting an act: failure to pay a bill

There are situations where failure to perform an act has a significant consequence. In such cases C would be consequated by a $T \rightarrow$ or an $A \rightarrow$ or by another (sometimes external) party. For example, if a phone bill is not paid by the end of time T,

the phone company will shut off service. $\begin{bmatrix} ab \\ C_4^{b+} \\ bA_3 \end{bmatrix}$ the phone company will shut off service. $\begin{bmatrix} bA_3 \\ bA_3 \end{bmatrix}$ the phone company will shut off service. $\begin{bmatrix} bA_3 \\ bA_3 \end{bmatrix}$ the phone company will shut off service.

The legend indexed to the subscripts might read:

- 1. If phone company a threatens to cut off service after time T.
- 2. Service would be cut off.
- 3. (If) customer b pays the phone bill within time T.
- 4. Service would continue.

The consequence C of the termination, with all of its modifiers, can be shown at the end of the terminating arrow.

4.8. Reciprocal vertical arrows: decision making and competition

When each of two consequences prevents the occurrence of the other, as in decision making and certain types of competition, reciprocal vertical arrows are required (see diagram).

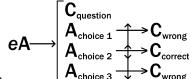
$$\begin{bmatrix} aA_1 & & \\ & aA_2 & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & \\ &$$

(Mechner, 1959) that prevents the essential elements of the diagram's structure from being obscured.²¹

The diagram on the left represents a making a decision. The reciprocal vertical arrow is needed to show that either decision precludes the other. The diagram on the right shows a and b competing to achieve different consequences (as in any zero sum game). Here, the reciprocal vertical arrow is required to show that once a has achieved C^{a+} , b can no longer achieve C^{b+} , and vice versa. If the vertical arrows were omitted, the occurrence of either $A \rightarrow C$ would not prevent the subsequent occurrence of the other.

4.9. Simultaneous multiple discrimination: multiple choice

The term multiple discrimination refers to the behavioral effects of contingencies in which the consequences of different acts (choices) depend on the prevailing stimuli (signals) and on the party's history with respect to those signals. The choices can be presented simultaneously or sequentially. When



taking a multiple choice test,

the student may be confronted with a question C together with simultaneously available choices (acts), as in the above diagram. The external agency e that presents (consequates) the C may be a teacher, a computer, or the student himself turning a page. The reciprocal vertical arrows show that each of the three acts terminates the availability of the others.

4.10. Two interrelated time intervals: the roast in the oven

The diagram shows a situation where a put a roast in the oven and left the house without turning the oven off (aA_1) . The roast will burn (C_5^{a-}) , unless the oven is turned off (A_8) within time T_4 .

$$aA_{1} \rightarrow \begin{bmatrix} aC_{2} & aC_{3}^{a+,ap} \\ T_{4} & & \\ aA_{6} \cap T_{7}) \rightarrow bA_{8}^{ap} \end{bmatrix} C_{5}^{a-}$$

If the oven is turned off (A_8) after T_7 and before T_4 , the roast will be done. But the oven will get turned off only if a asks (aA_6) her neighbor b to do so before T_4 . T_7 and aA_6 therefore have an "and" relationship.

Legend:

 aA_1 : If a leaves the roast in the oven on leaving home

 ${}_{a}C_{2}$: The roast would be in the oven with the oven on.

 T_4 : Time after which the roast would burn.

 T_7 : Time after which the roast would be done.

 C_5^{a-} : Burnt roast.

 aA_6 : If a calls her neighbor b and leaves her a message

 $_{ab}C_9$: Message to turn off the oven after time T_7 .

 $bA_8 \rightarrow$: If b turns off the oven after T_7 and before $T_4 \dots$

 C_3^{a+} : The roast is done and C_5^{a-} is averted.

The b in abC_9 's lower left quadrant implies that b got the message. The ${}^{a}p$ attributes of A_{8} and C_{3} represent a's

²¹ The unabbreviated way would be to show two separate vertical arrows that cross each other, each one emanating from one of the two originating events and cutting the horizontal arrow of the other.

prediction/estimation of the probability that b will actually do bA_8 and that the roast will therefore be taken out in time.

4.11. Types of "or" relationships: achieving priority, putting out a fire

One kind of "or" relationship is where either one of two or more acts can result in a given consequence. Another is where a single act can result in either one of two or more consequences. Both can in turn be divided into exclusive (either, or, but not both) and inclusive (either, or, or both) "or" relationships.

Diagram on the left: an exclusive "or" relationship shown by a merging of the horizontal arrows. The consequating entities in such diagrams can be As and/or Ts. This diagram could represent two parties competing to consequate C, where whoever gets there first obtains the only available C (as when parties compete for priority in applying for a patent or in reaching the South Pole).

Diagram on the right: either one of two parties or both together can put out the fire—the inclusive "or," represented by the union symbol \cup .

 $(aA_1 \cup bA_2) \rightarrow (aA_3 \cap bA_4) \rightarrow C^{ab+}$ would be read as, "If either a or b or both (note the inclusive "or" symbol \cup) create a situation in which cooperation between a and b (note the \cap symbol for "and") can occur, the consequence would be C^{ab+} ."

4.12. Probabilities of alternative consequences: Russian roulette

Another type of "or" situation is where a single act *A* (e.g., the tossing of dice) consequates two or more possible outcomes with different probabilities. In "Russian roulette" there is a 1/6 probability of the consequence "bullet in chamber" occurring. If the focus is on the two alternative scenarios, the arrow can be shown as branching, with the other branch consequate.

ing the complementary probability 5/6. A $C^{+, 5/6}$ In general, contingencies involving alternative consequences are represented by a multi-pronged fork with two or more arrows pointing to the different possible C_s .

The analyst may sometimes wish to show that a modifier like *perceive* and *predict*, or a valence, has an "if" in front of it. For example, he may want ${}_{a}C$ to be read as "If a would perceive C" rather than the standard "a would perceive

C." He would then show the two possibilities
$$A \xrightarrow{a p C} a^{p} C$$

as the branches of an "or" fork. For example, when considering whether or not to commit a crime (Becker, 1976, 1995, 1997) a party might weigh the consequences of the crime being perceived (p) or not perceived (1-p) by society. (For a fuller analysis, see Mechner, 2008).

4.13. Sequential multiple discriminations: traffic lights, naming people

In a sequential multiple discrimination, signals appear one at a time.

Example. We stop when the traffic light is red and proceed when it is green, and call different people by their respective names.

To codify this type of contingency we first ask how the circumstances for the acts are consequated. Traffic lights are controlled by timers, and telephone equipment transmits the voices of people we may want to name when they call us. In both cases external agencies *e* consequate the situations in an "or" manner—the traffic light is either red or green, and we receive phone calls from one person or another.

$$eA \xrightarrow{\begin{array}{c} A_{\text{stop}} \\ C_{\text{red}} \end{array}} C_{\text{stopped}}$$

$$eA \xrightarrow{\begin{array}{c} C_{\text{green}} \\ A_{\text{go}} \end{array}} C_{\text{moving}}$$

Again, the "or" relationship is represented by a fork. When the multiple discrimination involves more than two signals, the fork would have correspondingly more branches.

4.14. Misperception of C: mistaken identity

If policeman a sees a suspicious character b, he may try to arrest him. If b, claiming mistaken identity, reached into his pocket (bA_{reaches}) for his identification (abC_{ID}) , then in T seconds this might take, the policeman could misperceive the visual consequence $(\mathbf{a}^{\mathbf{x}}\mathbf{C}_2)$ of (bA_{reaches}) as pulling out a gun, and shoot b dead, mispredicted by b. The \tilde{b} term means that b would not predict a's misperception.

$$\begin{bmatrix} a \mathbf{C}_{\text{suspic. charact.}} & b \mathbf{A}_{\text{reaches}} \\ a \mathbf{A}_{\text{arrest}} & b \mathbf{A}_{\text{reaches}} \end{bmatrix} \xrightarrow{\tilde{b}_{ax}} \mathbf{C}_{2} \xrightarrow{a \mathbf{C}_{\text{dead}}} \mathbf{C}_{\text{ID}}$$

4.15. Misperception of a missile test

A similar diagram could describe a misperception by country a in response to a missile test by a hostile power b, and the associated mispredictions by b.

$$bA_{\text{tests}} = \begin{bmatrix} \tilde{a} & \tilde{c} \\ h^{x} & \tilde{a}^{x} & \tilde{c} \\ h^{x} & \tilde{c}^{x} & \tilde{c}^{x} \\ h^{x}$$

Here, \tilde{a}_{test} shows a's non-prediction and misperception of the test. If a responds with aA_{attacks} , the consequence would be negative for both parties. The b^x in the upper left quadrant of the a^x shows that b would mispredict a's misperception of the test. The b^x in the upper left of A_{attacks} shows that b would mispredict the attack by a. If it were desired to show that b would not predict rather than mispredict a's misperception, \tilde{b} would replace b^x in a^{x} 's upper left quadrant.

Many types of situations that involve misperception, and misprediction of misperceptions, are of great concern to societies and nations, as well as in interpersonal relationships. Behavioral contingency analysis helps identify the critical variables whose effects can then be studied by various methods.

4.16. Concatenated probabilities in systems analysis: the Trojan horse

If Homer had been the analyst, he might have shown Odysseus – the general of the besieging Greeks – as party a scheming the following contingency:

"If we build a giant hollow wooden horse and leave it for the Trojans to find (aA_{offers}) , the Trojans (b) may misperceive the horse (as being empty rather than filled with soldiers). If the Trojans thereupon take the horse into their city ($bA_{\text{takes in}}$), our soldiers hidden inside the horse may be able to emerge from the horse during the night (aA_{emerge}) and open the gates for the rest of us to enter and sack the city, thereby winning the war."

$$aA_{\overline{\text{offers}}} \xrightarrow{\tilde{b}} \begin{bmatrix} \tilde{b} \\ a_{(b^{x})^{a}p_{1}} \\ bA_{\text{takes in}}^{ap_{2}} \\ bA_{\text{takes in}}^{ap_{2}} \\ a_{\tilde{b}} \end{bmatrix} A_{\text{emerge}}^{ap_{3}} \xrightarrow{a_{b}} C_{\text{sacked}}^{b, a+, a_{p_{4}}}$$

The b in the upper left quadrant of C_{horse} shows Odysseus's bet that the Trojans would not predict this "gift." The b^x in the lower left quadrant of C_{horse} shows b's possible misperception, and the ${}^{a}p_{1}$ in b^{x} 's attribute quadrant shows Odysseus' prediction/estimation of the probability of this misperception. Odysseus might also have predicted/estimated the probabilities of various subsequent events— ap_2 for the act $bA_{\text{takes in}}$, and ap_3 for aA_{emerge} (e.g., the Trojans might discover the ruse, or the horse's door might jam). The \tilde{b} symbols in the two verb quadrants of aA_{emerge} show that b would not perceive or predict aA_{emerge} . The attributes quadrant of C_{sacked} shows that this consequence would be negative for b, positive for a, and that a would have predicted/estimated its probability as p_4 .

If Odysseus had indeed performed this analysis and considered the product of the four probabilities, he might have concluded that his scheme was very (too?) risky.

4.17. Vigilance²² and the analysis of interrelated probabilities

If (p_1) e plants a bomb (eA_{plants}) that has probability p_7 of going off after unknown time $T^{M?}$, and if $(aA_{\text{vigilance}})$ shows that

a is probably (p_2) engaged in vigilance/detection activity, then a will detect (perceive) ${}_{a}C_{\text{bomb}}$ with probability p_{3} . If a thereupon alerts (aA_{alerts}) the bomb squad b (probability p_4) with a signal (abC_{alert}) , the bomb squad will probably (p_5) attempt to disarm the bomb (${}_{b}A_{\text{disarm}}$), with p_{6} being the probability of doing so successfully.

$$\left(e\mathsf{A}^{\rho_1}_{\mathsf{plants}} \cap a\mathsf{A}^{\rho_2}_{\mathsf{vigilance}}\right) \rightarrow \begin{bmatrix}\mathsf{T}^{M?} & & & \\\mathsf{disarmed} \\ & & \mathsf{C}_{\mathsf{bomb}} \\ & & \mathsf{d}\mathsf{A}^{\rho_4}_{\mathsf{alerts}} & & \\ & & & \mathsf{ab}\mathsf{C}_{\mathsf{alert}} \end{bmatrix}^{\rho_5} \underbrace{\mathsf{C}^{\rho_5}_{\mathsf{explosion}}}_{\mathsf{ab}\mathsf{C}_{\mathsf{alert}}}$$

4.18. Modification of contingencies: mitigating a danger

This diagram shows a situation in which two or more parties cooperate to avert a threat. Again, C^{ab-} can come after an unpre-

dictable time $T^{M?}$ with probability p. $(aA \cap bA)$ By acting cooperatively $(aA \cap bA)$ (e.g., by exercising vigilance, building levees, or storing provisions), a and b can prevent C^{ab-} .

To show that $(aA \cap bA)$ merely reduces the probability of C^{ab-} from p_1 to p_2 , rather than to zero, the changed contingency would be shown at the end of the vertical arrow with the new probability p_2 , as in the diagram.

$$\begin{array}{c}
\mathsf{T}_{2}^{M?} \to \mathsf{C}^{ab - p_{2}} \\
\mathsf{T}_{1}^{M?} & \to \mathsf{C}^{ab - p_{1}}
\end{array}$$

$$\begin{array}{c}
\mathsf{C}_{aA \cap bA}
\end{array}$$

 $\begin{bmatrix}
\mathsf{T}_{1}^{M?} & & \\
(a\mathsf{A} \cap b\mathsf{A}) & & \\
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&$ both the timing and the probability of the terminated contingency. If the new contingency requires a bracket, the arrow would point to the bracket.

4.19. Risky choices: thinking ahead in a game

The risks that a chess, checker, or go player takes when choosing between moves are the uncertainties regarding (1) how accurately or completely he identified the opponent's possible responses, (2) which of these the opponent will actually choose, and (3) the valence of the outcomes of these possibilities.

If a considers two possible moves aA_1 and aA_9 , and thinks two moves ahead in considering b's possible responses, a would be uncertain as to whether, in response to aA_1 , b would choose bA_3 (say, a particular identified move) or bA_5 (say, a possible

$$aA_{1} \xrightarrow{\uparrow} bA_{3} \xrightarrow{\uparrow} C_{4}^{2^{a}}$$

$$bA_{5} \xrightarrow{\downarrow} C_{6}^{2^{a}}$$

$$bA_{7} \xrightarrow{\uparrow} C_{8}^{2^{a}}$$

$$bA_{10} \xrightarrow{\downarrow} C_{11}^{2^{a}}$$

unidentified move), to aA_9 , b would choose bA_7 (say, another particular identified

²² Experimental studies of vigilance and observing behavior and the associated contin gencies include Dinsmoor (1983), Dinsmoor et al. (1972), Case and Fantino (1989); and Nevin et al. (2005).

move) or bA_{10} (say, another possible unidentified move). Furthermore, a would be uncertain regarding the valences of the resulting situations (note the a s in the attribute quadrants of the question marks).

4.20. Divergent contingencies for two parties: a sexual overture

If a makes a sexual overture aA_1 to b, the two possible outcomes, from a's point of view, are that b would accept (probability p) and that b would decline (1-p). The reciprocal vertical arrow is needed to show that b's accepting and declining

$$aA_1 \rightarrow \begin{bmatrix} abC_{\text{overture}} \\ bA_{\text{accepts}}^p \\ bA_{\text{declines}}^{(1-p)} \end{bmatrix} \xrightarrow{ab} C_{\text{acceptance}}$$

are mutually exclusive.

From *b*'s point of view the issue is whether to accept or decline, rather than a probability issue. This type of examination of two divergent perspectives can be useful in fields like law, business (see Section 6.7), economics, or public affairs, where the differences in the contingencies for each party can be important.

4.21. Game theory situations and bluffing: blackmail and kidnapping

If blackmailer a threatens to harm b unless b pays, the probability of the harm actually occurring, even if aA_{executes} , might be p_2 . If aA_{demand} stated that b can avert harm by bA_{pays} , then the probability that a would do no harm if b pays is p_4 , and

ah
$$C_{\text{new demand}}$$
 ab $C_{\text{new demand}}$ the probability that a

would actually execute the threat if a does not pay is p_1 , that is, a is bluffing. The probability that $bA_{\rm pays}$ would consequate a recycling of the entire contingency is p_3 . When a contingency recycles, all components of that contingency are restored for the next cycle, including ones terminated by vertical arrows in the preceding cycle.

The same diagram could also describe hostage taking or kidnapping. This example and many of those that follow illustrates how behavioral contingency analysis can serve as a tool for identifying the variables and their relationships, including utility, that would be considered in the types of economic analyses proposed by G.S. Becker (1976, 1992, 1995).

4.22. Predictions/estimations of p: the basis for a wager

Here aA^{M1} and bA^{M2} represent bets that a and b can place in situation abC_5 on, say, the outcome of a ball game. The external agency e could be two ball teams producing one of the two possible consequences.

$$e\mathsf{A}_{3} \rightarrow \boxed{(a\mathsf{A}_{1}^{M_{1}} \cap b\mathsf{A}_{2}^{M_{2}})} \xrightarrow{a_{b}\mathsf{C}_{4}^{a+,b-,(a,b)}p_{1}} \\ a_{b}\mathsf{C}_{5}^{a-,b+,(a,b)}p_{2}}$$

The attributes M_1 and M_2 of the bets can be the amounts of money a and b would be betting. The symbol \cap indicates the "and" relationship of the bets, as in a contract. The notations ${}^{(a,b)}p_1$ and ${}^{(a,b)}p_2$, with commas between the a and b, show that a and b have different predictions/estimations of p_1 and p_2 (they would not be betting unless they did). If there were no comma, just ab, it would mean that a and b have a joint prediction/ estimation.

4.23. Perceptions of M: tipping a service provider

If customer b has received a service (aA_1) from a (say, a waiter), the magnitude M of the tip b gives (bA^M) may be M_1 , M_2 , or M_3 . The three tips are C_3 , C_7 , and C_{11} , and a may have subjective/idiosyncratic perceptions of their magnitudes.

$$aA_{1} \xrightarrow{bA_{2}^{M_{1}}} \begin{bmatrix} aC_{3}^{\text{Ref.} \#1} & \text{Ref.\#1:} (a^{x})^{bp_{1}}M_{1} \\ aA_{4} \xrightarrow{ab}C_{5}^{b,bp_{4}} & \text{Ref.\#2:} (a^{x})^{bp_{2}}M_{2} \\ bA_{6}^{M_{2}} \xrightarrow{a} aC_{7}^{\text{Ref.} \#2} & \text{Ref.\#2:} (a^{x})^{bp_{2}}M_{2} \\ bA_{8}^{M_{3}} \xrightarrow{ab}C_{12}^{b+,bp_{13}} & \text{Ref. \#3:} (a^{x})^{bp_{3}}M_{3} \end{bmatrix}$$

The magnitudes are attributes of the three Cs and the idiosyncratic perceptions of the magnitudes (indicated by a^xs in the Ms' lower left quadrants), are shown in the three references, a technique used because if these attributes were scaled to the size of the main diagram and inserted, they would be hard to read.

The fact that the two sets of three Ms have the same (lower right) subscripts means that they represent the same amounts of money. (The three-way reciprocal vertical arrow shows that these three possible consequences are mutually exclusive.) The legend might elaborate that a would idiosyncratically perceive M_1 as too small, M_2 as acceptable, and M_3 as generous. These three idiosyncratic perceptions have probabilities p_1, p_2 , and p_3 , respectively, which b might predict/estimate, as shown by the bs in the upper left quadrants of the three probabilities.

a's possible reactions aA_4 and aA_9 would consequate C_5 or C_{10} , which b would perceive and whose respective probabilities b would predict/estimate as p_4 and p_5 . C_5 might be an insult and C_{10} might be an expression of gratitude.

These and many other intricate social and cultural variables at play in many tipping situations have been discussed by Green et al. (2003), and a brief review of the relevant literature is presented in Lin (2007).

4.24. Misperception of the M of a C: perpetration of a fraud

If seller a offers (aA_1) to sell b a counterfeit painting, C_3 , a would perceive the value of the painting to be M_4 (as indicated by the a in the lower left quadrant of C_3 , which is distributive to C_3 's attribute M_4),

138
$$aA_{1} \rightarrow \begin{bmatrix} a_{b}C_{3}^{a}b^{x}M_{4} \\ bA_{2} \rightarrow bA_{4} & bC_{5}^{b-1} \\ a_{b}C_{6}^{(a+,b-)M_{7}} \end{bmatrix}$$
its value, indicated by the b^{x} in M_{4} 's

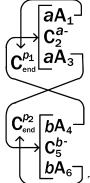
while b would misperceive

its value, indicated by the b^x in M_4 's lower left quadrant. Note that both a and b perceive the painting, but only b misperceives its value. The a's in the two left quadrants of the b^x indicate that a would perceive and predict b's misperception of the value, which is what makes the transaction a fraud. The description of aA_1 in the legend might include history events responsible for b's misperception.

If b accepts a's offer aA_1 and buys the painting (bA_2) , paying a the asking price M_7 (an attribute of C_6 's valence), then a+ would be a receiving the M_7 and b— would be b parting with it. If bsubsequently gets the painting appraised (bA_4) and consequently learns its true value ${}_{b}C_{5}{}^{b-}$, the valence of that information would be negative for b. If a third party c witnessed the fraud and stood to benefit from it, c would face the choice of warning b, or letting the fraud occur, thereby becoming an accomplice.

4.25. Symmetrical reciprocal recycling contingencies: feuding

This diagram represents a feud where parties a and b alternate



hurting each other. The crossing diagonal recycling arrows show the symmetry of this type of contingency. Attempts to end the feud can be made at any point by A_1 or A_6 (peace overtures) that have probabilities p_1 and p_2 of being effective.

Other examples of contingencies that have symmetrical recycling structures: bargaining, where two parties take turns making each other offers and counteroffers; situations where two parties take turns assisting each other; or a chess game that uses a chess clock where each player's move A^t stops his own clock and starts the other player's. If a player exceeds his limited allotted time, he loses (see Mechner, 2008).

4.26. Mutual deterrence and first strike

Mutual deterrence is another type of symmetrical recycling contingency rife with game theory dynamics that can occur in military confrontations and other situations where there can be standoffs. Two factions, a and b, both have the option of launching a first strike. If a attacks first, b will retaliate unless a's attack terminates b's ability or willingness to do so. The protagonists will usually differ in their perceptions and predictions/estimations regarding the magnitude of any damage inflicted as well as the various probabilities that (a) the other party will execute a first strike, (b) that there will be retaliation for each attack, (c) that the first strike will avert retaliation, and (d) that any given retaliation will end the cycle of exchanges. Each party would also consider the other party's prediction/estimation of its own predictions/estimations of the various probabilities and magnitudes, and the predictions regarding possible misperceptions, mispredictions, and uncertainties regarding the modifiers of these entities at several levels up the recursion hierarchy (see Mechner, 2008).

These types of complex interactive situations can be addressed by operations research or computer simulation, with behavioral contingency analysis helping to identify and confront relevant variables that could easily be overlooked.

5. Behavioral contingency analysis as a research tool

5.1. Widely studied contingencies: schedules, the "matching law"

The basic contingencies for the widely studied fixed or variable interval schedules and fixed ratio schedules (Skinner, 1938) would be codified as $\rightarrow T \rightarrow A \rightarrow C^+$ and $\rightarrow nA \rightarrow C^+$, respectively, where the *n* represents the number of consecutive As rather than the agent of A (Mechner, 1959). 23

In the contingency shown at the right, there is a limited

 $A \rightarrow C$ time window T_1 during which $A \to C$ can recycle $T_1 \to T_2$ (e.g., "Make hay while the sun shines"), or be allowed to occur only once (termed "limited hold" in Ferster and Skinner, 1957). T_2 is the parameter that distinguishes a free operant situation from a trial situation. The usual possible modifiers of the As, Cs, and Ts can suggest independent variables in experimental analyses of this general contingency.

Another widely studied contingency is the one to which the "matching law" applies (Herrnstein, 1970; Baum, 1979), which states the observed relationship (often hyperbolic, e.g., Mazur, 2001) between measures of behavior and measures of consequences. The diagram represents a generic "matching law" experiment—one that studies behavior allocation or other manifestations of choice as a function of the magnitude or probability of consequences. The contingencies represented by this diagram are seen in most situations that involve choice or decision making, and in many that are the subject of behavioral economics research (Herrnstein et al., 1997; Camerer and Lowenstein, 2003; Becker, 1976, 1995, 1997).

²³ The number n could also be shown as an attribute of the A. Either notation is an abbreviation for the notation that shows the A as recycling n times with the consequation of C^+ depending on the C register (see Section 6.2 (C)) having reached n.

$$\Rightarrow aA_1 \Rightarrow aT_2 \Rightarrow aA_3 \Rightarrow aC_6^{a_{(+)}M_8, p_{10}}$$

$$\Rightarrow aA_1 \Rightarrow aT_2 \Rightarrow aC_4$$

$$\Rightarrow aA_5 \Rightarrow aC_7^{a_{(+)}M_8, p_{11}}$$

C₄ represents the two manipu-

landa or physical facilities for the two choices A_3^t and A_5^t . The t attributes of the choices can refer to the amount of time allocated to each of the two choices or to their rates, inter-response times, or latencies. M_8 and M_9 represent the magnitudes of the valences of the two consequences (e.g., amount of reinforcement), and p_{10} and p_{11} represent their probabilities (e.g., reinforcement probability or frequency). A_1 can be a changeover response (as when a switches from A_3^t to A_5^t or vice versa) and T_2 is the amount of time the changeover takes up.

5.2. Temporal discounting research

Temporal discounting research deals with the relationships between the behavioral effects of the magnitude of a consequence and its delay (e.g., Commons, 1987; Lattal, 1987; Mazur, 1987; Critchfield and Kollins, 2001; Green et al., 2004, 2007; Rachlin, 2006). In many situations, less/sooner can be equivalent to more/later for certain values of time and consequence magnitude, with the equivalence contour often described by a hyperbolic function (e.g., Mazur, 1987). The trade-off between delay and magnitude of reward in this type of research has sometimes been associated with "delay of gratification" or "self-control" (Rachlin, 1995).

The diagram below is a generic representation of temporal discounting experiments that would use real, as opposed to verbally described, delays and consequences.

One delay-magnitude

combination is T_4/M_{14} , and the other is T_8/M_{15} . The two choices are aA_2 and aA_3 . The diagram provides a road map for identifying this research area's potentially operative variables besides the basic ones of delay, valence magnitude, and probability of the consequence. For instance:

- C_6 and C_9 signal the two delay intervals (which can be "filled or empty," MacInnis, 2007).
- The acts aA₅^t and aA₇^t indicate the possibility of filling the delay intervals with alternate behavior maintained by various possible contingencies.
- The \pm valences show that the consequences can be positive or negative.
- The probabilities p_{15} and p_{16} of the consequences are shown as a possible alternative or supplement to the valence magnitude variable.
- All of the time intervals and consequences are shown as modifiable by perceptions and predictions that could be a function

- of signaled or experiential history variables (e.g., number of repetitions).
- Not shown in the diagram but also possible variables are misperceptions or mispredictions and non-perceptions or nonpredictions of any of the entities.

The diagram also highlights the issue of using, as the independent variables, real delays and consequences (as Mazur did with rats and pigeons, Mazur, 1987) versus verbally described (sometimes termed "hypothetical") consequences and/or time delays (e.g., Kowal et al., 2007; Johnson and Bickel, 2002). When the independent variable is embodied in a verbal statement, ${}_{a}C_{1}$, presented simultaneously with the choice contingency, and the consequences of the choices are also verbal statements, C_{4} and C_{5} , the diagram reduces to the one at the right. In this type of experiment, a description of ${}_{a}C_{1}$ in the legend would explain

the experimenter's assumptions regarding the subjects' behavioral and verbal histories with respect to the described delays and consequences. A key issue here would be the assumed correlation between the behavioral effects of (a) real delay/consequence combinations and (b) verbal descriptions of these, given certain verbal histories. Some of these issues are discussed in Hertwig and Ortmann (2001), Johnson and Bickel (2002), and Fantino et al. (2007).

5.3. Research on the prisoner's dilemma

This class of behavioral contingencies, which is of sweeping importance in the social sciences, is very well explained in the Wikipedia article entitled "Prisoner's dilemma," (http://en.wikipedia.org/wiki/Prisoner's_dilemma). The central theme of such situations is the decision on the part of a party to "cooperate or defect" (Rapoport and Chammah, 1965). This dynamic is present to some extent in many social situations, especially choice situations that relate to short-term versus long-term self-interest (sometimes termed "selfish versus altruistic") (Rachlin et al., 2001), in situations involving predictions as to choices that others will make (e.g., Yi and Rachlin, 2004), and in many types of game-theoretic situations that involve social acts like voting in elections, coalition formation (Gelman, 2003), or behavior related to global warming (e.g., Clemons and Schimmelbusch, 2007).

The generic prisoner's dilemma diagram that follows shows four combinations of a and b cooperating or defecting, with four possible consequences.

$$(a^{ab}A^{ab}_{coop} \cap b^{ba}A^{ba}_{coop}) \xrightarrow{(ba,ab)} C_3^{a(\pm)M_4,b(\pm)M_5,p_6} C_3^{ab}A^{ba}_{coop} \cap a^{ab}A^{ba}_{coop} \cap a^{ab}A^{ba}_{def}$$

$$(a^{ab}A^{ba}_{coop} \cap a^{ab}A^{ba}_{def}) \xrightarrow{(ba,ab)} C_3^{a(\pm)M_{10},b(\pm)M_{11},p_{12}} C_3^{a(\pm)M_{10},b(\pm)M_{11},p_{12}}$$

$$(a^{ab}A^{ab}_{coop} \cap b^{ba}A^{ba}_{def}) \xrightarrow{(ba,ab)} C_{15}^{a(\pm)M_{10},b(\pm)M_{17},p_{18}} C_{15}^{a(\pm)M_{10},ab} C_{15}^{a(\pm)M_{10},b(\pm)M_{17},p_{18}}$$

$$(b^{aa}A^{ba}_{def} \cap a^{ab}A^{ab}_{def}) \xrightarrow{(ba,ab)} C_{21}^{a(\pm)M_{22},b(\pm)M_{23},p_{24}}$$

$$n_j$$

140

Every act and the probability of every act is subject to prediction by the other party, and every possible consequence and its M and/or p for each party can be perceived and predicted by both parties, with all predictions being subject to further prediction by the other party at an indeterminate number of levels up the recursion hierarchy. Not shown in the diagram (due to graphical limitations) is that every prediction can also be a misor non-prediction, and can be further modified by an uncertainty on the part of the predicting party.

As in the case of temporal discounting research, behavioral contingency analysis of these types of situations helps identify the numerous operative variables that can be studied experimentally or used in simulations, and that can easily escape notice absent such analysis. Some of these are:

- The number of times the contingency has recycled with resulting learning history effects, as seen in studies that showed stabilization of choice behavior (e.g., Thomas and Feldman, 1988).
- Effects of other types of experimentally imposed learning histories.
- Different numbers of recyclings for different choices and consequences.
- Differences in the possible positive or negative consequences for each party (the consequences need not be symmetrical for the parties).
- The effects of the magnitudes of the valences, which can also be asymmetrical.
- The probabilities of the consequences, as distinct from those pertaining to the choices, that is, the same sets of choices need not always result in the same consequence, or in consequences that have the same modifiers.

Also, as in the case of temporal discounting research, this analysis highlights the issue of using "real" versus verbally described consequences.

5.4. Specifying independent variables in neuroscience research

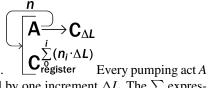
Some neuroscience experiments on the mapping of neural functions require an operationally specified independent variable that is under the experimenter's control. Behavioral contingencies meet this requirement (See Mechner, 2008).

6. Progressive changes and registers

6.1. Pumping water into a bucket

There are important contingencies in which a consequence keeps changing as a function of recyclings or other variables. The codification of such contingencies requires the use of a register that shows the changing magnitude of the consequence.

This diagram describes the water level in a bucket being



filled using a hand pump. $\ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \$ Every pumping act A increases the water level by one increment ΔL . The \sum expression in the C_{register} 's magnitude position (attribute quadrant) shows the amount of water in the bucket after every n_i such As, calculated as the cumulative number of times (n_i) the A has recycled at every point times the change in the water level ΔL with each cycle.

6.2. Other registers of progressive changes



This diagram shows the generic representation of contingencies that involve progressive changes as a function of $i, f(n_i, t_i)$.

Examples.

- (A) When $f = \sum (n_i/t_i)$, the register represents the rate at which act *A* would be occurring at any given value of *i*, for a desired range. *Instances*: response rate in experiments, proficiency assessment when practicing a skill.
- (B) When $f = \sum (t_i/n_i)$, the register could show the act's average duration after every n_i repetitions, within a desired range. *Instances*: indicator of speed when learning to do a job faster or practicing a skill, keeping track of the average time between contractions during labor.
- (C) When $f = \sum (n_i)$, the register shows the total number of acts that have occurred at every value of *i. Instances*: points scored in a game, money saved up so far, number of votes so far during election coverage.
- (D) When $f = \sum_i (t_i)$, the register shows the total elapsed time value of *i. Instances*: watching the clock, concern that a job is taking too long.
- (E) Exponential functions could show progressive effects of various kinds of growth or proliferation, of exponential decay, or of other asymptotic functions.

6.3. Short-term and long-term contingencies: global warming

When sustained repetition of a type of act has a cumulative long-term negative effect at individual, societal, or national levels, such acts are often maintained by their short-term positive consequences. There are also acts whose sustained repetition has short-term negative and long-term positive consequences (e.g., IRA savings accounts, doing aversive work to accumulate money).

The diagram at the right shows some of the behavioral dynamics of global warming—the cumulative consequence

of many individual acts like coal burning or burning of vegetation that are maintained by their short-term positive consequences with possible resulting long-term temperature change.

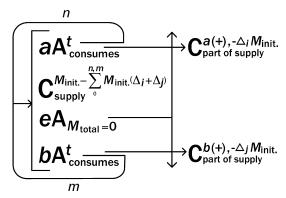
temperature register In this diagram, there is no identified end point of the process, unlike in the case of filling a bucket with water. The register $C_{\text{temperature register}}$ is shown as having the magnitude M of its negative valence (again represented by the formula in the exponent position) being cumulatively and progressively incremented as a function of the number i of repetitions. This type of analysis can help identify the roles of some of the variables that may affect global warming (e.g., Clemons and Schimmelbusch, 2007).

Other familiar examples of this same contingency structure are the possible long-term effects of smoking and the entire gamut of addictive behaviors, of consuming excessive sugar on health, or of dumping chemicals and wastes into waterways. All such contingencies involve repetitive behavior with positive short-term and negative long-term consequences, and differ only in the descriptions of the As and Cs and of the variable that is changing (e.g., Δh for health or $\Delta temp$ for temperature, as above). In all these cases, the $C_{register}$ would show the cumulative effect at every interim point.

6.4. Competitive depletion of a limited supply

If two parties concurrently and incrementally consume a limited supply, the ultimate consequence is depletion of the supply.

Examples. Cats lapping milk from the same bowl, overfishing in a body of water, overlogging a forest.

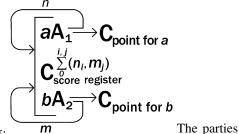


In the diagram, the initial magnitude $M_{\rm init}$ of the limited supply $C_{\rm supply}$ would be progressively reduced by the amounts consumed by the two parties—a representing one party and b all the others. Every n_i th of a's n consumption acts reduces the supply by an additional $\Delta_i M_{\rm init}$, and each m_j th of b's m consumption acts reduces the supply by an additional $\Delta_j M_{\rm init}$. ²⁴ The t attributes of the consumption acts can represent rates or dura-

tions. When the M of the supply has thus reached zero or some other threshold, the external agency e terminates $(eA_{M=0})$ the consumption cycles. The description of the effects of this contingency can be tuned by the usual *perceive* and *predict* modifiers of the three Cs and by the values of their Ms or ΔM s.

6.5. Registering points in a game: keeping score and communicating it

In many kinds of games, like basketball, soccer, or football, the winner is the player or team that scored the most points, often by the end of a certain time period. In the diagram, every aA and bA represents the scoring of a point, and the register $C_{\text{score register}}$ shows the scores of the two teams during the game. The score, shown by the summation formula, is a magnitude dimension



of the $C_{\text{score register}}$. m perceiving it are not shown in the diagram.

6.6. Feedback regarding progress in races

Most games, races, or contests end when one of the parties has crossed a finish line, scored more points than the other party at the end of time T, or has met some other definition of the goal whose achievement determines the winner. The contestants' acts that consequate progress toward the goal can consist, for example, of a runner's strides or a rower's oar strokes. The two progress registers would show each contestant's progress after any particular number of such acts, as the summation of all progress increments up to that point. The pre-subscripts of the progress registers would show whether each party has access to their own and/or each other's progress indicators, information that is usually important in such contingencies.

Examples of races or contests where each party knows only its own state of progress. Two research teams racing to be first to publish an important discovery; two corporations competing to be first to bring a new product to market (see Mechner, 2008).

6.7. Divergent points of view: loans and insurance

When a loan reaches its term, the interest payment cycle is terminated and the borrower b either repays the loan or defaults. From lender a's point of view, these consequences would have probabilities p and 1-p. In the case of a default, a then has a choice between several acts, each having a different consequence. From b's point of view, the possible consequences of a's decision process would have probabilities p and 1-p. A register could display the accumulated interest payments as the loan approaches maturity (see Mechner, 2008).

 $^{^{24}}$ The term M could obviously be any function defining the magnitude of the change at each decrement.

A similar example is an insurance contract, which usually involves the payment of insurance premiums on a time cycle analogous to that for interest payments. Here, a register for the accumulated premium payments reflects the perspective of the insurer. The diagram for the full contingency (see Mechner, 2008) shows an external agency e that unpredictably generates possible mishaps against which the beneficiary is insured. The beneficiary's point of view shows the possible acts of the insurer in case of such a mishap (e.g., paying the award, disputing the entitlement, undertaking a study of circumstances), as alternative consequences with different probabilities, and the insurer's point of view shows them as a choice or decision.

6.8. Elections

In elections, the campaigning acts of each party are assumed to consequate votes for it. A summation term in the two registers could show the presumed number of votes each party has accumulated up to any point in time. The winner is determined by the difference in the number of votes accumulated by the two parties by Election Day (see Mechner, 2008).

7. Applying behavioral contingency analysis

7.1. The value of behavioral contingency analysis

The behavioral contingency language can be used to address practical problem situations in diverse fields of human affairs. If the behavioral outcomes of a situation are important – potentially very valuable or very threatening – the time and effort involved in analyzing the contingencies may be justified.

In practical applications, the analyst's goal would usually be to gain a better understanding of the contingencies that are at play in the situation so as to increase his ability to modify them and thereby come closer to achieving the desired behavioral outcomes. The result of the analysis may reveal that the contingencies are different from what they were thought to be or that there are some at play that were not noticed, especially when the situation is complex and involves multiple parties. A systematic behavioral contingency analysis may also expand the list of the parties that could be considered, their possible acts and the consequences of those acts, the probabilities and magnitudes of those consequences, and the parties' relevant perceptions, predictions, misperceptions, and mispredictions regarding aspects of the situation. In research applications, the value resides in a clearer and more complete understanding of the behavioral contingencies, and in their objective and operational specification when they constitute independent variables.

7.2. Using prompts as an algorithm

The prompts listed below constitute an algorithm for guiding the process of performing a behavioral contingency analysis of any situation. These prompts help the analyst register every relevant thing he knows or assumes about the situation being analyzed. Each prompt is understood to be qualified by the words "where relevant to the analyst's desired focus." In practice, the

answers to most of the listed prompts would not be relevant to the desired focus and therefore not needed.

- Identify and list the parties involved in the situation.
- For each party, indicate its possible act(s) A.
- For each act, indicate its possible consequence C.
- Indicate the valence of each C for each party.
- Indicate the magnitude *M* of each valence.
- For each act (where relevant) indicate a duration variable.
- For each C, indicate its probability p.
- For each *p*, indicate the party(ies) that might have a prediction/estimation of it.
- For each C, indicate the time lags T.
- For each T, indicate any A or T that may change or terminate it
- For each *C*, indicate any *A* or *T* that might prevent or change it.
- For each termination, indicate the replacing *C* or contingency.
- For each p, indicate any A or T that might change it.
- For each A, C, agent of A, and their modifiers indicate the party(ies) that perceive, predict, misperceive, mispredict, or do not perceive or predict it.
- For each *M*, indicate the party(ies) that misperceive or idiosyncratically perceive it.
- Indicate all As and Ts that may initiate recycling arrows, and the termination points of these arrows.
- For each recycling arrow, indicate n where it has significance.
- Define any consequence that changes as a function of n.
- If the change in that quantity has an end point, define that end point, and the consequence C of it being reached.

When responding to prompts that involve time, magnitude, or probability, the analyst may initially just note the variable, and only when known would he indicate actual or relative values.

Just going through the process of responding to these prompts would tend to improve an analyst's understanding of the operative contingencies, the parties to be taken into account, and the factors and variables to be considered. The analyst would also be prompted to identify and confront areas of ignorance or uncertainty about the situation.

7.3. The role of behavioral science

The contingency structures that an analyst produces when analyzing any given situation²⁵ would flow directly from the information, beliefs, and assumptions he registered and entered

²⁵ In practice, large and intricate diagrams are difficult to display in two dimensions. The analyst can bypass the need to do so by using the prompt method, which permits full use of the language's hierarchical/recursive features to express any desired nuance without the constraints of printed diagrams. The creation of more complex diagrams will require computer software that can convert the registered information into multi-dimensional behavioral contingency structures. While these would exist only in the computer memory, two-dimensional diagrams of particular portions could be printed out for use in checking and debugging. The development of software with these capabilities is in progress at the time of writing.

in response to the prompts. To predict the resulting behavioral outcomes, he would draw upon his knowledge of the behavioral effects of various types of contingencies, of various types of behavioral histories, of signaling positive or negative consequences having various relative magnitudes and probabilities, of cues regarding contingencies, and of time factors. The contingency diagrams themselves obviously do not embody such knowledge and therefore cannot generate predictions of behavioral outcomes. Such predictions require the application of knowledge provided by the behavioral sciences.

The analyst can iteratively revisit and readjust the contingency information he entered in response to the prompts when he sees a discrepancy between the predicted behavioral outcomes and his expectations. The modification and design of contingencies by this general method could become one of the practical uses of contingency analysis in operations research, computer simulation, and the management of behavioral contingencies in any area of human affairs. As the database of knowledge provided by the behavioral sciences continues to expand, the usefulness of behavioral contingency analysis can be expected to increase correspondingly.

7.4. The pedagogic value of behavioral contingency analysis

The sometimes heard comment that behaviorism has limited relevance to complex human behavior or to practical concerns can be answered by pointing to the sweeping applicability of behavioral contingency analysis. Students who learn to use the present behavioral contingency language, and make efforts to apply it, quickly become persuaded of the great and general relevance of behavior analysis to human affairs.²⁶

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A PowerPoint tutorial on the behavioral contingency language and its applications can be dowloaded from the website http://www.mechnerfoundation.org. Estimated completion time: 3–15 h, depending on time spent on the exercises. The course received a dry run at AILUN University in November of 2007 under the auspices of Prof. Giulio Bolacchi.

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