CHESS AS A BEHAVIORAL MODEL FOR COGNITIVE SKILL RESEARCH: REVIEW OF BLINDFOLD CHESS *BY ELIOT HEARST AND JOHN KNOTT*

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This multifaceted work on chess played without sight of the pieces is a sophisticated psychologist's examination of this topic and of chess skill in general, including a detailed and comprehensive historical account. This review builds on Hearst and Knott's assertion that chess can provide a uniquely useful model for research on several issues in the area of cognitive skill and imagery. A key issue is the relationship between viewing a stimulus and mental imagery in the light of blindfold chess masters' consistent reports that they do not use or have images. This review also proposes a methodology for measuring and quantifying an individual's skill shortfall from a theoretical maximum. This methodology, based on a 1951 proposal by Claude Shannon, is applicable to any choice situation in which all the available choices are known. The proposed "Proficiency" measure reflects the equivalent number of "yes–no" questions that would have been required to arrive at a best choice, considering also the time consumed. As the measure provides a valid and nonarbitrary way to compare different skills and the effects of different independent variables on a given skill, it may have a wide range of applications in cognitive skill research, skill training, and education.

Key words: chess research, mental imagery, conceptualization, cognitive skill measurement, skill training, representations, visual perception

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Perhaps the reason why chess so often fascinates psychologists is that along with mathematics, music, and other arts, it has produced displays of virtuosity sometimes viewed as pinnacles of human achievement. One of the more dazzling feats of this sort is blindfold chess—chess played without sight of the chessboard or pieces, with the players calling out their moves. Hearst and Knott show us why this topic should interest not only chess players but also behavior researchers, neurobiologists, psychologists, and educators.

Eliot Hearst straddles the worlds of chess and psychology at the highest levels. By age 21 he had already achieved national and international prominence as one of the most talented senior chess masters of his generation. He is pictured in the 1952 photo (Figure 1), standing, with the three other members of the Columbia College chess team of which he was the captain. In 1962, he captained the United States Olympic Chess Team.

Hearst did his graduate work at Columbia University under Professor W.N. Schoenfeld and then achieved prominence once again but this time as a behavior researcher (Hearst, 1979, 1988; Hearst, Besley & Farthing, 1970; Hearst & Jenkins, 1974) while a professor at the University of Missouri, Indiana University, The University of California at Berkeley, Columbia University, and The University of Arizona. The other author, John Knott, is a lifelong researcher and prominent authority on blindfold chess.[1]

The book clearly stands as the definitive compendium on the topic of blindfold chess. Chess players have already expressed admiration for the depth of its scholarship,[2] including the authors' painstaking and masterful analysis of some 444 historically significant blindfold chess games, and psychologically flavored biographical sketches of history's greatest blindfold chess masters.

Behavior analysts will be particularly interested in the chapters "Research on General Chess Skill" and "Psychological Studies and Commentaries on Blindfold Chess" (pp. 151–190),[3] which review the salient research literature on those topics, including the work

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[1] For more information on the authors, see www.blindfoldchess.net.

[2] The book has been reviewed very favorably in 18 publications and won the Fred Cramer Award for Best Chess Book of 2009.

[3] Page numbers in the citations refer to the book being reviewed.

Fig. 1. The Columbia College chess team of 1949–1952 after a radio match with Yale. Right to left: James Sherwin, Eliot Hearst, Carl Burger, Francis Mechner (Courtesy of the Columbia University Archives).

of Alfred Binet, Alfred Cleveland, several Russian psychologists, Adrian de Groot, Herbert Simon, William Chase, Dennis Holding, Fernand Gobet, Neil Charness, Christopher Chabris, and many others, including Hearst himself.

Chess as a Unique Research Model

The book draws attention to a number of issues in the realm of cognitive behavior that have received little attention in the behavior analytic literature, possibly for lack of a methodology with which to address them (Foxx & Faw, 2000; Marr, 2003; Staddon, 2001). The features of chess that should make it interesting to behavior analysts as a model for cognitive behavior research are these: The choices (chess moves) are discrete, involve purely cognitive behavior, are susceptible to registration and quantitative evaluation by computer, and the number of choices available in a given position (approximately 37 on average) is convenient. These features may be some of the reasons why Hearst and Knott believe (pp. 150–151), as does former world chess champion Garry Kasparov (2010), that chess can serve as a useful laboratory model for cognitive skill research. Chess has, in fact, been dubbed "the drosophila of cognition research and psychometrics" (Chase & Simon, 1973; Van der Maas & Wagenmakers, 2005; p.150).

The present review attempts to show how the above-listed features of chess provide a research methodology applicable to a wide range of problems whose investigation requires quantitative measurement of skill and knowledge.

Notable Findings

In their review of the literature, Hearst and Knott document a number of notable conclusions that may surprise chess players and nonchess players alike:

- The general memory of chess masters, including those able to play many blindfold games simultaneously, is no better than that of the average person.
- Highly skilled players can form long-term memories of full-board chess positions within seconds of viewing them.
- High level chess skill (not just blindfold chess) requires a recognition-action reper-

toire of some 50,000 to 100,000 features of chess positions and associated responses.

- Some of the strongest masters find the actual sight of a chess position to be more distracting than helpful when thinking ahead during a game.
- Practicing blindfold chess improves sighted chess skill.
- Some of the strongest blindfold chess masters claim that the strength of their blindfold play is similar to that of their sighted play.

But the bombshell, which prompts the present reexamination of "visualization," is this: *blindfold chess masters consistently report that what they visualize are not images of pieces or chessboards, but abstractions of these with minimal or no physical features.* A typical report is, "I do not visualize real pieces but I know where they are."

Chess Masters' Protocols Regarding Their "Visualizations"

The fourteen or so blindfold champions quoted by Hearst and Knott describe what they do in these terms: "no mental pictures," "abstract knowledge," "I know where the pieces are," "only an abstract type of representation," "only relationships," "no real picture," "the significance of a piece," "knowing what combination or plan is in progress," "lines of force," "pieces are only friend or foe, carriers of particular actions," "sort of formless visions of the positions," and so forth. Many of the masters report that they have no mental image at all (p.151).[4]

Such introspective reports and protocols regarding "private" events and processes generally tend to be accorded low status as behavioral data,[5] but here may be an instance where such protocols, given their consistency, number, and relative clarity, need to be taken into account.

Several of the blindfold champions also explain that what is essential for blindfold chess skill is fluent knowledge of the color and name of each of the chessboard's sixty-four squares, and the lengths and intersection squares of all the diagonals. Hearst and Knott add that, "Geometric knowledge of the chessboard ... presumably underlies what an expert blindfold player means when he talks about visualizing "lines of force" or "powers of a piece"—rather than seeing actual pieces and colored squares in the mind's eye."

Viewing and Visualizing

These reports point to the need for a detailed examination of the relationship between *viewing*, in the sense of contemporaneous perception of external stimuli with light falling on the retina, and the behavior generally referred to as "visualization," or "mental imagery." What do the two types of behavior have in common and how do they differ?[6] Defining visualizing as "internal seeing" or "seeing a stimulus in its absence" (Moore, 2008; Skinner, 1953, 1974) does not address the issue of what behavior is involved in either viewing (i.e., retinal seeing) or visualizing, or how they differ.

These are the most obvious differences between viewing and visualizing:

- Viewing involves the retina while visualizing does not.
- Viewing is associated with a contemporaneous exteroceptive stimulus while visualizing is not.
- Viewing permits the stimulus to be scanned and interrogated regarding even its most unimportant details (e.g., the colors, shapes, sizes, or surface characteristics of the chess pieces, the source or level of the illumination), while visualizing does not.

The Image Conjecture–an Illusion

"Image conjecture" is the term I am applying to the widespread belief that one can "have an image" in the sense of a reproduction, copy, retrieval, or reconstitution of the image's optical attributes (Kosslyn, Thompson & Ganis, 2006), without actually viewing an exteroceptive visual stimulus. This view may be based in part on the faulty reasoning that retinal images can be "transmitted" to the brain in full detail and then "seen" (by whom?) (Bennett & Hacker, 2003;

[4] This reviewer, who himself had a chess master rating and has played simultaneous blindfold chess, would describe his "visualizations" in the same terms as those reported above, and can corroborate from personal knowledge that most chess masters who can play blindfold chess (and most can) would report likewise.

[5] For a discussion of the conceptual issues concerning the status of introspective verbal reports, see Locke (2009).

[6] This analysis bypasses the extensive vision literature (e.g., Bruce, Georgeson & Green, 2003; Marr, 1982, Noë, 2004), which deals with phenomena that are not directly relevant here.

Noë, 2004). It may also be based in large part on our subjective experience—the powerful introspective illusion that we "see" an internal image "in the mind's eye." When applied to chess, the Image conjecture would imply that blindfold masters "have" internal images of chess positions, but their protocols clearly state that they do not.

The proposed conceptualization may seem radical. It rejects the common premise implied in much of the psychology and neuroscience literature, that "mental imagery," "visual imagery," or "representation" is so elemental and self-evident as not to require further definition or examination. The alternative view offered here and developed in the sections that follow, is that "seeing" can occur *only* in the sense of viewing, where a visual stimulus can be scanned and interrogated, and cannot occur in "mental imaging," "visualization," or "seeing with the mind's eye," none of which permit scanning-interrogation. The proposed conclusion is that such "visualization" behavior actually consists exclusively of the recall of *previously acquired concepts drawn from our preexisting behavior repertoires.* Our subjective sensation of internally "seeing" this type of "mental image" is purely an introspective illusion whose compelling power may be due to a kind of fusion or blending phenomenon—perceiving continuity where the stimulus elements are, in fact, totally discrete. Other examples of this type of illusion are our perception of smooth continuous motion when we see the discrete still frames of a movie—the frames being analogous to the recalled concepts, or our illusion of a likeness of a face when viewing a digitized array of a relatively small number of square black, white, and gray pixels.

A Terminological Quandary

The present analysis thus requires a term that does not connote an internal or mental image. The terms most often used in the psychological and neuroscience literature are *visualizing, mental imaging* and *representation,*[7] all of which have this undesired connotation. The term should also distinguish between viewing and visualizing. The terms *conceptualization* or *abstraction* don't make that distinction because they refer to behavior that occurs in both.

This quandary reminds us that when a scientific discipline's traditional terminologies cannot accommodate a new requirement, one must choose between redefining an old term and coining a new one, a common occurrence in the evolution of disciplines (Mechner, 2008, pp. 236–237; Zentall, Jackson-Smith, & Jagielo, 1990). Imaging research may now be at this point.

The terms that fully circumvent the inherent dualistic mentalism of the Image conjecture are, paradoxically and ironically, *mentalization* and *mentalize.* Saying that blindfold chess masters mentalize chess positions avoids the implication that they "have a mental image," because it permits the (parsimonious) interpretation, in conformity with the chess masters' protocols, that only conceptual behavior occurs in mentalization. The term mentalization has the added virtue of being applicable to all modalities—visual, auditory, tactile, olfactory, or kinesthetic.

Mentalizations Consist of Concepts

The present formulation is based on the behavior-analytic concept of "concept," defined as "discrimination between classes and generalization within classes" (Hull, 1951; Keller & Schoenfeld, 1950).[8] Thus, a conceptual triangle may be outlined or not, filled in or not, of any color or size, right, acute or obtuse, verbally defined, or a compound of all of these attributes; the concept of a familiar person may include features, clothing, facial expressions, or a compound of all of these; similarly, when chess masters mentalize a chess position, they conceptualize certain of its relational and dynamic attributes.

The range and diversity of concepts (including "abstract" ones, and chained concepts referred to as skills) are as great as our behavior repertoire itself, which includes all of our knowledge and memories, and these may be linked to different sensory modalities

[7] The term "representation" has been used in a multiplicity of senses in cognitive neuroscience—isomorphic mappings (copies) of mental images, the reproduction or retrieval of an image (Huk, 2008), and conceptual or abstract responses (Martin, 2007). The term confounds these usages, while our need here is to distinguish between them. And it is uncomfortable to apply the term "image" to other sensory modalities, as in "auditory imagery" when referring to music or a voice (Smith, Reisberg, & Wilson, 1992).

[8] The term *concept* is used here in the broad sense that regards skills, whether motor or cognitive, as chains of concepts (Mechner, 1967).

or combinations of them. Concepts linked to the visual modality include not only the basic hard-wired "vocabulary of vision" concepts (edges, brightness, colors, contrasts, movements, distance to and between objects, directional location, etc.; Zeki, 1978; Zeki et al., 1991), but also the multiple synthetic complex human counterparts of the chick's innate concept of the overhead silhouette of a hawk, and range from fast-learned concepts (e.g., the human face) to more slowly learned ones like written words. Perhaps the richness of our repertoires of visual concepts contributes to the illusion of an internal image—the illusion that may render somewhat counterintuitive the notion that mentalizations can be accounted for fully by concepts that are drawn exclusively from the existing behavior repertoire.[9] An incidental methodological bonus of viewing mentalization as consisting entirely of such concepts is that doing so brings it into the purview of the behavioral disciplines related to conceptualization and learning, including equivalence research, relational frame work, skill training, etc.

The Scanning-Interrogation Process in Viewing

There is ample evidence (from tachistoscopic and other studies) that subjects retain quasi-photographic images somewhat like after-images, known as iconic memory, for approximately a quarter of a second after exposure of a visual stimulus (e.g., Sperling, 1960). During that brief time, such images can be scanned and interrogated almost as if they were viewed stimuli. This biological phenomenon may have its evolutionary roots in locomotion behavior, which necessitates short-term recall of the features of terrain being traversed, and may also play a role in reading, listening, scanning, etc. (Mechner, 2009).

The scanning and interrogation behavior that occurs in viewing involves eye movements that focus successively on iconically retained small-areas, largely those subtended by the fovea. The cues that guide eye movements to successive focus areas are provided mainly by peripheral vision (Blackburn and Nguyen, 2002). The iconic memories of the cascading succession of small focus areas permit them to be blended into mosaic images sufficiently complete and coherent to be perceived as meaningful stimuli (Koch and Ullman, 1985). Without iconic memory, such blending would not be possible.

Some features of such viewed images may be retained in longer-term memory as concepts when (a) attention is somehow directed to them, (b) an instruction or other type of contingency generates the behavior of scanning for a particular feature, or (c) an encountered stimulus feature is recognized as significant, based on a learning history (e.g., Bichot, Rossi, and Desimonel, 2005). Those are cases in which a stimulus feature may be conceptualized and retained in longer-term memory.

Conceptualization in Viewing and Mentalizing

Some conceptualizations can occur in either viewing or mentalizing, and some (e.g., "abstract" or "verbal" ones) can occur only in mentalizing. Conceptualizations that can occur in viewing when scanning and interrogating the stimulus might take such forms as, "A fly just landed on the tip of the brown cow's white left ear," or "You are frowning." If these same conceptualizations were to occur in mentalizing, they would not be the result of scanning—they would be drawn from the behavior repertoire. In chess, if the conceptualization "Queens are still on the board," occurred in viewing a position, it would result from scanning and interrogating the chessboard; if it occurred in mentalization, it would be imported from the repertoire.

Some concepts have physical embodiments that can be viewed, and some do not. A concept like "car," for instance, has a physical embodiment, but the fact that it can be viewed from the inside or the outside, front or side, near or far, makes it a concept that could also be mentalized as an abstraction. Concepts that do not have physical embodiments and therefore cannot be viewed (e.g., time, quantity, love), though usually termed abstract, are still concepts in the behavioral sense of the term.

[9] It is also worth noting in this connection that mentalizing is not limited to the recall of previously perceived stimuli. Most chess positions mentalized in blindfold play or when thinking ahead in sighted play (chess players usually call this "calculating") have not been seen before. Similarly, we can easily mentalize novel sentences, melodies, or scenes that we have never actually seen or heard. That is because the concepts that occur in mentalization can consist of recombinations, rearrangements, or syntheses of features, components, or other attributes of previous conceptualizations—visual, auditory, emotional, relational, or abstract ones. Much of what we call thinking, too, may consist of recombination, reassembly, and syntheses of such components into novel configurations (Bar, 2007; Mechner, 1994, pp. 10–11).

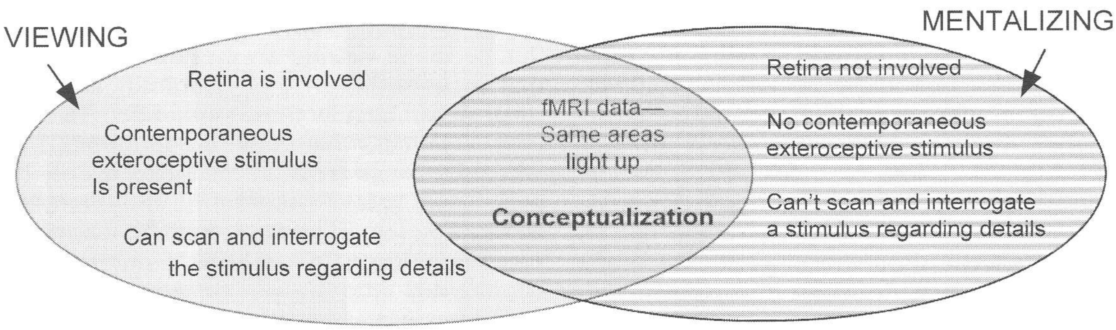

Fig. 2. The relationship between viewing and mentalizing.

Important in the context of conceptualization that occurs in viewing is the fact that concepts already in the behavior repertoire can, and often do, override contemporaneous perception (Brown, 1973; Carter & Werner, 1978; Cumming, Berryman, & Cohen, 1965; Mechner 1994, pp. 33–34; Schoenfeld & Cumming, 1963; Wright & Cumming, 1971)—what we perceive is largely a function of what we have learned to perceive (Graham, 1951, pp. 911–915; Skinner, 1953; Woodworth & Schlosberg, 1955, pp. 403–491). Thus mentalizations that occur in viewing can override the reality of what is actually there, a phenomenon that explains many instances of misperceptions and mistakes not only in chess but also in all kinds of everyday situations. This phenomenon may also explain the fusion or blending effect that may be responsible for the ''mental imaging'' illusion.

What Viewing and Mentalizing Have in Common

Many fMRI studies show that viewing and mentalizing activate similar brain areas, suggesting that they involve at least some of the same behavior (Borst & Kosslyn, 2008; Kolb & Wishaw, 2009; Kosslyn, 1980, 1994; Richardson, 1999; Wheeler, Peterson & Buckner, 2000).[10] For that behavior we need look no further than conceptualization. The similarity of the activated areas has sometimes been misinterpreted as evidence for the Image conjecture (Edelman, Grill-Spector, Kushnir & Malach, 1998; Kourtzi & Kanwisher, 2001), but the parsimonious interpretation is that it is only the conceptual behavior that viewing and mentalization have in common. The Venn diagram in Figure 2 shows this relationship.

The Image Conjecture as an Empirical Proposition

Although the Image conjecture is adequately refuted by the scanning-interrogation test, it can also be formulated as the empirical proposition (unsupported though it may be) that the iconic memory images on which viewing depends can be retained for hours or days rather than just split seconds, and can be scanned and interrogated during that longer time much like a viewed stimulus, a kind of ''photographic memory'' effect, as in ''eidetic imagery.'' But the extreme rarity of abilities like those exhibited by Kim Peek (Treffert & Christensen, 2005) and some others (Treffert, 2009) (whose behavior when mentalizing resembles the scanning and interrogation of a complex visual stimulus like a page of print), makes it unlikely that such a phenomenon is involved in normal instances of mentalization. In addition, most mentalized chess positions (as well as nonchess mentalizations) are novel, whereas eidetic imagery applies to previously viewed stimuli. Finally, the masters' protocol data are inconsistent with the possibility that their mentalizations involve eidetic imagery.

Alleged Evidence for the Image Conjecture

The Image conjecture and introspective illusion may also drive some common misinterpretations of imagery research data. Many of the studies often cited as supporting the Image conjecture, some of which are reviewed by Hearst and Knott (2008, pp 166–178), and by Kolb and Wishaw (2009, pp. 639–645), are open to alternative, more behavioral interpretations. For example: A measure like time

[10] The same findings have also been reported for audition (Intons-Petersen, 1992; Naatanen, 1985; Reisberg, 1992; Smith, Reisberg, & Wilson, 1992) and for olfaction (Bensafi et al., 2003).

needed for the mental rotation of a shape or for finding a place on a recalled map does not differentiate between accessing an "internal image," and history-based conceptual processes. The reason is that time required for a mental rotation task can be interpreted as time needed to complete a purely conceptual routine—one that was learned via a history of actual physical rotations of objects, with associated observation and conceptualization of the stimulus changes corresponding to each fractional amount of rotation. The same type of alternative interpretation would apply to the Kosslyn, Ball, and Reiser (1978) finding of time–distance correlations in the reading of mentalized maps. Nor can eye movements linked to spatial or geometric aspects of a mentalized stimulus (e.g., Donahoe & Palmer, 1994, pp. 253–256) be interpreted to mean that an image is being scanned, because eye movements, as well as other motor or verbal behavior, are often coordinatively linked to conceptualization behavior that has spatial or geometric aspects. Palmer (2010, pp. 38–39) makes the related point that more research attention should be directed to eye movements.

Charness (1976, p.159) found that most experienced chess players can form robust long-term memories of chess positions after viewing them for only a few seconds, and reconstruct them much later after having spent the intervening time in activities designed to be interfering. But in interpreting this finding it is important to note that the masters retained only the ability to reconstruct the positions, not necessarily images of them, and that this ability can be based on conceptual behavior exclusively.

The "Recognition-Action" Repertoire of Concepts

A large repertoire of piece configuration concepts is evidently needed for both blindfold and sighted chess (p. 9, 91, 127, 190). Chase and Simon (1973, pp.157–158) estimate that master-level play requires a "recognition–action" repertoire of 50,000 to 100,000 such concepts (also called "chunks," "templates," or "patterns").[11] The "action" in such a recognition–action concept can be a single move or a whole sequence of moves, a plan, an algorithm, or the recall of the behavior of another (or the same) player in a similar position. The blindfold masters' protocols, (e.g., "knowing which combination or plan is in progress") suggest that the action can consist of a sequence of moves as a unit (pp. 151–152, 162–166; Chase and Simon).

Mentalizing in Normal Chess

Many strong chess masters report that they actually find it helpful to look away from the board when thinking ahead ("calculating") (pp. 9, 127, 151, 189–190). The reason for this could be that when considering a move in normal over-the-board play, the player typically tries to mentalize the position that will exist after the move is made while at the same time still viewing the conflicting stimulus of the about-to-be-moved piece in its original position.

But why would the helpfulness of looking away depend on skill level? Because when looking away, the player relinquishes sight not only of the about-to-be-moved piece but also of all the other pieces—the ones that would not move. Sight of these others would be helpful only to the extent of the player's uncertainty regarding their placement when looking away from the board. The stronger the player, the smaller that uncertainty, the less help is derived from the sight of them, and therefore the greater the extent to which the interference effects outweigh the value of viewing them and the board.

The Matching of Mentalizations

Relevant to Hearst and Knott's discussion of Chase & Simon's (1973) "recognition–action" thesis is the suggestion that skilled performances (of which chess is an example) are normally practiced and honed by matching a mentalized model (rather than a contemporaneous exteroceptive one) of the desired performance (Mechner, 1994, pp. 29–34). For instance, musical performers, especially when practicing, try to match their mentalization of the music as they want it to sound, and a golfer may try to match his mentalization of Tiger Woods' swing. Chess players, too, sometimes strive to match mentalized model behavior as when they covertly ask themselves, "What would a grandmaster (or my coach) do in this position?" or "What did I do in a similar position in a previous game?" What is matched in such cases is a mentalized behavioral event, either one that actually occurred (a recalled event), or one that never occurred, as when a dancer mentalizes, say, a slinking

[11] Behavior analysts would conceptualize it as a multiple discrimination repertoire of that magnitude.

tiger—a composite of conceptualized elements.

The Progression from Novice to Expert

Hearst and Knott report de Groot's 1965 conclusion (p.54), later supported by others, that stronger players consider roughly the same number of moves as weaker players, but consider better ones, and faster. The explanation may be the same as the one offered in the motor performance literature for the changes that occur as a performance progresses from novice to expert due to learning and practicing: Performance components that functioned independently and separately in the novice performance become linked and integrated into increasingly effective functional units, but the number of such functional units does not change—only their internal makeup and organization changes. A summary of the literature regarding this process, which has been described as the development of "coordinative structures" (Belen'kii, Gurfinkel & Pal'tsev, 1967; Bernshtein, 1967; Normand, LaGasse, & Rouillard, 1982; Turvey, Fitch, & Tuller, 1982; Turvey, Schmidt, Rosenblum, & Kugler, 1988) is provided in Mechner (1994, pp. 39–47). That is how expertise develops not only in motor skills like tennis but also in nonmotor skills like chess or problem solving, where the key components are covert or cognitive (Gobet & Charness, 2006).[12]

"Mental Practice" and the Benefits of Blindfold Play

The benefits of practice accrue to the particular skills that are practiced—to motor skill when motor skills are practiced, and to covert or cognitive skills when it is those that are practiced (Mechner, 1994, p. 39). That is why "mental" practice is most beneficial in performances that have significant cognitive components, and least beneficial in performances that depend mainly on overt motor behavior (p. 170; Heuer, 1985; Ross, 1985; Ryan, Blakeslee, & Furst, 1986; Ryan & Simons, 1983; Van Gog, Paas, Marcus, Ayres, & Sweller, 2009). Since sighted chess is at the cognitive extreme of the motor-cognitive range and depends heavily on mentalization skill, it should be expected to benefit significantly from playing blindfold chess, which amounts to pure mentalization practice. This would explain and support the claim by grandmasters and Hearst and Knott that blindfold play improves sighted play.

A Skill Measurement Methodology

Many types of skill research require a dependent variable that quantifies a choice's shortfall from best. Hearst and Knott cite as examples problem solving in mathematics, physics, architecture, music, sports, and financial decision making, all of which involve mentalization in the consideration of alternatives (p.149).

The proposed methodology assigns operational meaning to the concept of a shortfall in relation to a specified theoretical maximum skill level, using chess computers to define best moves in given positions.[13]

Hearst and Knott (pp. 163–164, 189–190) describe the use of this approach by Chabris and Hearst (2003) who used the chess computer Fritz 5 to rate a move's shortfall from "best" using pawn-equivalents as the arbitrary units of measurement. They used these ratings to compare blindfold and sighted play for the frequency and magnitude of "mistakes" (there was no significant difference), defining "mistake" as a 1.5 "pawn" shortfall[14] from best, and "blunders" as significantly larger shortfalls.

The Issue of a Measure's Arbitrariness

In view of the fact that every chess position must result in a win, loss, or draw when played out to the end of the game, any rating of the strength of a move or the magnitude of a mistake is necessarily arbitrary. Computer-generated ratings reflect the computer's evaluation of the position after it has

[12] Experienced chess players can cite numerous examples of such larger units and linked actions. Examples: When the configuration is a queen's side pawn majority, consider strategies that will convert that majority into a passed pawn; in certain positions, assign high priority to the placement of a rook on an open file or on the seventh rank, especially when doing so includes a threat. For expert players, such algorithms are unitary concepts perceived in a split second. The authors cite studies that discuss the thousands of hours of study needed to acquire the necessary number of such unitary concepts and linked elements for the achievement of master-level skill.

[13] The sense in which the term "best" is used here is simply that even in the occasional instances where there exist still stronger moves, little would be gained by including them, even if they could be found. When there are several such moves in a given position, any one of them would qualify as "best."

[14] A "pawn-equivalent shortfall from best" can be thought of as a measure of how much better or worse a particular move makes the position, using the value of a pawn as the unit of measurement.

calculated ahead a certain number of moves. But how many moves and moves of what strength? Because the answers to these questions are arbitrary, so are such ratings.[15] If ratings are arbitrary, they do not permit meaningful comparisons for different values of an independent variable, either within or between skill areas. To be usable for such comparisons, the units of measurement must have the same meaning regardless of the independent variable used and the skill area involved.

Traditional Skill Rating Methods and Their Limitations

Skill or knowledge involving choice behavior (henceforth referred to simply as "Skill") is often studied with "right-or-wrong" items (Chase & Ericsson, 1982). Problems of the widely used "what's-the-best-move" type are normally scored either as "percent correct" or with someone's subjective ratings of the possible answers. But a wrong choice does not provide information about the nature or magnitude of the responsible Skill deficit, and an overall score does not identify the items that presented problems.

Hearst and Knott make frequent reference to the numerical Skill ratings that reflect players' past competitive performance results against each other. Such ratings, widely used in competitive games like chess (p.143, 163)[16] and go,[17] being relative, are numerically arbitrary and have no anchorage points—they float and drift. If all rated chess or go players were to become stronger or weaker to the same degree at the same time, their respective ratings would not change.

Requirements of a Generally Useful Skill Measure

To be useful, a measure of chess skill should be applicable to the performance of an individual player facing particular positions. The measure should quantify the magnitude of players' Skill deficits and pinpoint their nature, separately for different phases of the game and for different types of positions. The measure proposed here meets these requirements and can be used to study the effects of such independent variables as training method, sleep, or practice techniques.

Measuring Skill Shortfall from a Theoretical Maximum

Claude Shannon (1951) in his paper "The Prediction and Entropy of Printed English" showed how a normal speaker of English can function as a human measuring instrument to quantify informational properties of letter sequences. He proposed the "entropy"[18] measure to quantify a subject's information deficit for each successive letter in such sequences. The $\log_2$ of the number of tries needed to identify each letter, expressed as "bits of information," corresponds to the number of yes–no questions the subject would have needed to ask if he had used a yes–no questioning strategy to identify the letter.

The method proposed here flips Shannon's (1951) procedure around, so that the entropy measure is applied to the *subject's Skill deficit* for each stimulus situation rather than to an attribute of the stimulus. Here a stimulus attribute is used as the known standard against which the Skill deficit is measured. The same method is obviously applicable to any multiple-choice situation in which all the possible choices, including the best one, are known and can be made available.

The Uncertainty Measure Applied to Choice Situations

The underlying rationale of the proposed procedure is that an individual's performance in any choice situation can be expressed in terms of uncertainty regarding the best answer. As in Shannon's (1951) procedure, Uncertainty ("U") is defined as the $\log_2$ of number of tries a particular individual would need to get the best answer, and is measured

[15] It might also be noted that the information such a rating conveys to a particular player depends on the player's skill level. For instance, a top grandmaster might interpret an advantage of 1.6 pawn-equivalents as an easy win, and a relatively inexperienced player might interpret it as a minor edge.

[16] The competitive-performance-based rating formula internationally used in chess, devised by the Hungarian physicist Arpad Elo (1978), arbitrarily assigns a rating somewhat below 1,000 to beginners, while players of world championship strength would usually achieve ratings in the vicinity of 2,800.

[17] In go, the competitive-performance-based rating scale used by the American Go Association for amateur players assigns a rating of around −350 (35 kyu) to rank beginners and +800 (8 dan) to the strongest amateurs. Professional players are rated internationally on a separate scale from 1 to 9.

[18] Or information deficit, disorganization, disorder, unpredictability, or its inverse: negentropy, degree of organization, or predictability (Wiener, 1948, 1950).

in bits. This U corresponds to the number of yes–no questions the individual would need if a question-asking procedure were applicable. Here is the underlying reasoning: Suppose you knew that a coin was tossed three times. To eliminate your uncertainty as to how it landed each time, you would need to ask three yes-no questions: (1) ''Did it land heads on the first toss?'' (2) ''Did it land heads on the second toss?'' and (3) ''Did it land heads on the third toss?'' That makes eight possible outcomes, and the $\log_2$ of 8 is 3. Thus, your uncertainty U was 3 bits.

The unique property of the U measure is that it is nonarbitrary—its units are always bits of information, regardless of the skill area. It is therefore applicable to a wide variety and range of skills, knowledge, and choice situations, and thereby enables quantitatively meaningful comparisons among these. Arbitrary measures, in contrast, do not permit valid comparisons of the type one normally wishes to make.

The U Measure Applied to Chess Skill

One of the reasons chess provides a useful laboratory model of choice behavior is that the Shannon (1951) procedure is readily applicable to it. Computer chess programs can define maximum Skill level operationally in terms of the best move (or the set of best moves when there is more than one) for any given position.[19]

The procedure: The subject is presented with a chess position on a computer screen and is instructed to indicate the best move. After each try, the computer responds ''correct'' or ''try again.'' The computer registers the number of tries needed to reach a best move for that position (without counting repeats).

For the hypothetical player who would always be correct on the first try, U would be zero bits because the log of 1 is zero—the Skill level at which a player would need to ask zero yes–no questions. A less skilled player, who might need, say, eight tries, would have a U of 3 bits (i.e., corresponding to three yes–no questions). So, the greater the Skill deficit, the greater the U.[20] The formula for U would thus be $U = \log_2 n$ where n is the number of tries needed to find the best move.[21]

A valid measure of ability to find best moves in given positions must reflect not just U but also time consumed, as a joint function of the two. Such a measure of achievement speed would be a useful dependent variable for studies in which the independent variable might be various types of training procedures, test conditions, types of positions, or player variables such as experience, Elo rating, fatigue, ingested substances, age, and so forth.

The Proficiency Measure

We will apply the term ''Proficiency'' to such a measure. We would want the theoretical maximum Proficiency score of a player who finds the best move on the first try in zero seconds to be 1.00 and to decrease toward zero as the number of tries n and the time consumed t increases. Thus both n and t must appear in the denominator. There are several formulas that meet these requirements but a straightforward one is $1/(1+ U \cdot t^k)$ where $U = \log_2 n$ as discussed above, t is the time used for the n tries,[22] and k is a scaling constant that sets the weight assigned to the time factor in relation to the U factor.[23] Thus number of tries can be traded off against thinking time in a way that leaves the Proficiency score unaffected—the player can think longer so as to require fewer tries, or use less thinking time by trying many choices quickly. The possibility of such tradeoffs means that a player's Proficiency score for each position need not

[19] A computer chess program named Rybka, reputedly the strongest in existence, plays at a strength level close to that of the strongest human grandmasters, and programs reputed to be even stronger have more recently been made available on the internet as free downloads. In the case of standard 19×19 go, on the other hand, there are as yet no computer programs that can generate moves beyond the level of weak players. However, there are now computer programs that can generate best moves for 9×9 go.

[20] A possible refinement of this procedure would give ''partial credit'' for trying moves that the computer considers second-best, third-best, etc. Such tries could, for instance, be counted as fractional rather than whole tries, with the amount of partial credit based on the computer's evaluation of move rankings, although any evaluation function is inevitably arbitrary.

[21] To reflect the effect of chance when the number of available choices is limited, as in chess where the average number of possible choices may be 37, n can be assigned an appropriate exponent (meaning that $\log_2 n$ would have a coefficient), so that U would approach its maximum of one at whatever faster rate may be desired.

[22] The computer software should subtract out the time consumed by the physical act of keying in each try.

[23] Other possible formulas for Proficiency that also meet the requirements are $1/(1+U \cdot e^{kt})$ where e is Euler's number, $1/(n^a \cdot t^k)$, and $1/(n^a \cdot e^{kt})$. The derivation of the term n^a is $e^U = n^{1.45}$, and the 1.45 exponent can be increased to a to reflect the effect of chance when the number of choices is limited, as discussed earlier.

be strongly affected by the amount of time spent—a "Proficiency constancy" that may accommodate stylistic differences between players in the way they use time.

Although Proficiency may often be correlated with Elo rating, it measures something other than practical competitive playing strength. Many players are weak in some types of positions and strong in others, or may collapse in a competitive stress situation but be brilliant at identifying best moves absent a competitive contingency. Such players may achieve high Proficiency scores but be weak in practical play.

Proficiency Constancy and the k Setting

The degree of Proficiency constancy depends on the k setting. If k is set at zero, time plays no role at all, and the Proficiency score would then be maximized by thinking as long as possible on each try so as to minimize n. On the other hand, if k is set very high, Proficiency would be maximized by trying every reasonable move as quickly as possible, as number of tries would then have little effect. In both of these cases, there would be no Proficiency constancy. Therefore, Proficiency constancy is maximized at some intermediate value of k.[24]

Here is one possible way to think about the k setting: Many players may need four to eight tries to find a best move, for a U in the range of 2 to 3 bits. If k is set at 0.2 (the fifth root) and t is measured in seconds, t to the k power would be 3 if the player spends 243 seconds (3 being the fifth root of 243) and 2 if the player spends 32 seconds. With that k setting, the time factor would have a relatively small impact on Proficiency—t would vary by a factor of around 8 (243/32), while t to the k power would vary by a factor of only around 1.5 (3/2). With higher settings of k, for instance 0.3 or 0.5, the time factor would have a correspondingly greater relative impact.[25]

In game applications like chess, k could be set low (corresponding to tournament conditions with long time limits), or high (corresponding to rapid play conditions). In skill areas other than games, such as timed tests, the k setting could correspond to a fluency requirement. The k setting also makes available a research methodology for ascertaining the effects on Proficiency of various time-sensitive variables. If k is scaled along the abscissa and Proficiency along the ordinate, a parameter would be represented by a family of curves. Examples of parameters could be age, rest, ingested substances, training variables, practice, or type of Skill.

The performance level of most chess players depends on the time limits used (e.g., rapid play versus play under standard tournament time limits), just as a runner's rating might differ for sprinting versus marathon running. The k setting is analogous to a time limit in that it sets the weight assigned to time in relation to U.

Addressing Questions Specific to Chess

There are also many chess-specific research questions that can be addressed with the Proficiency measure: Is the well documented peaking of strength in chess players' mid thirties (Elo, 1965) correlated with a peaking of Proficiency? The Proficiency measure can also be used to compare players from different countries or clubs whose standards, unanchored as they are, tend to drift apart over time. When applied over long periods of time the Proficiency measure would permit comparisons of players who may not even have been alive at the same time.

The Proficiency measure can also be used to investigate experimentally some of the claims and hypotheses cited by Hearst and Knott. For instance, is the chess masters' claim that their blindfold play is as strong as their sighted play matched by a corresponding similarity in their blindfold and sighted Proficiency scores?

Applications in Other Skill Areas Involving Choice

The methodology for measuring Proficiency is applicable to the study of any game or Skill domain defined by discrete time-constrained choices or decisions for which all available choices, including the best one, are known, as in chess. Examples are certain types of identification, classification, or problem solving tasks in business, legal, counseling, military, and social situations, and in academic subjects like spelling, geography, mathematics,

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[24] In developing the required software, a mathematically inclined programmer might consider using calculus of variations in programming the speed–accuracy trade-offs, and Lagrangian multiplier methods for the "optimality with constraints" problem—an optimization function that has been used in economics in connection with "utility functions," in operations research, and in solving optimization problems in control engineering.

[25] Since k would obviously have different effects in other possible Proficiency formulas, it would be assigned appropriately different values.

the sciences, certain verbal skills, literature, or social studies.

Many performance skills involve mentalization processes that are invisible to an observer (Mechner 1994, pp. 18–21), as when a chess trainer observes the trainee searching for a move. Data consisting of the trainee's incorrect tries could provide the trainer with a valuable window into such mentalization processes—one that would be more informative than the oral reports obtained in response to the "think out loud" type of request that chess trainers sometimes use (de Groot, 1965; Silman, 1991). The same technique would be applicable to the coaching of any skill that involves choice.

Applications in Skill Areas that Do Not Involve Choice

Since a key component of chess Skill is the ability to think ahead ("calculate"), which requires mentalization of positions that may occur several moves later, it follows that any improvement in mentalization skill would also be reflected in the Proficiency measure. Thus Proficiency could be regarded as an indirect measure of mentalization skill. This consideration becomes relevant in extending the methodology to Skill domains that do not involve choice behavior but that depend, as does chess Skill, on mentalization.

Consider, for instance, musical composition skill, which depends on auditory mentalization skills. Similarly, drawing from life depends in part on the ability to mentalize the model when the eyes are on the drawing rather than on the model. Unlike chess, neither of these skills involves discrete or definable choices. Although the procedure for measuring Proficiency would therefore not be directly applicable to them, it may nonetheless be plausible to extrapolate to them certain results obtained in chess Proficiency research. In the 1960s I tested a mentalization exercise that consisted of alternating several times between mentalizing a particular sequence of chess moves and actually seeing that same sequence played out on the board. If the Proficiency measure now showed that this type of exercise is, indeed, effective for improving chess mentalization skill, one might infer, by extrapolation, that similar types of exercises would improve mentalization skill in skill areas that don't involve choice, like musical composition skill and drawing from life. For example, musical mentalization skill might be improved by repetitive alternation between actually hearing a passage and mentalizing it. Similarly, the type of mentalization skill used in drawing from life or copying may be improved by repetitive alternation between looking at the model and mentalizing it.

Conclusion

This book can be expected to stand for a long time as the definitive compendium on blindfold chess for chess players, chess historians, and students of games. Many of the topics covered also have provocative implications for conceptual and research issues in behavior analysis, psychology, neuroscience, performance learning, training, and education. The present review explores these and describes a possible behavior analytic methodology for addressing them. While there are, as yet, few Skill areas in which quantitative measurement methods have been developed, the proposed methodology might stimulate such development. In addition to its potential uses in educational testing and training, it may also expand the methodological armamentarium of behavior analysis for the experimental study of cognitive behavior.

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